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Assignment
Sample paper
[2022-2023]
ODD SEMESTER

ASSIGNMENT

Reg No : 912020114002

Name : R. AKASH

Sub Code : ME 8593

Sub Name : Design of Machine Elements.

1. A link of S-shape made of a round steel bar is shown in Figure. Material for the link is steel with a yield stress of 380 MPa in tension. For a factor of safety of 4, find the diameter the steel bar.

Given data:

$P = 1.5 KN = 1500 N$

$R = 4d$

$\sigma_y = 380 MPa$

$n = 4$

To find

Bar diameter 'd'

Solution:

$M_b = 1500 \times 4d = 6000 d$ N-mm

$r_o = 4d + 0.5d = 4.5d$

$r_i = 4d - 0.5d = 3.5d.$

For round (circular) section, from PSG DB

$$r_n = \frac{(\sqrt{r_o} + \sqrt{r_i})^2}{4} = \frac{(\sqrt{4.5d} + \sqrt{3.5d})^2}{4}$$

$$r_n = 3.9843d$$

$$e = R - r_n = 4d - 3.9843$$

$$e = 0.015d$$

Bending stress.

$$\sigma_{b\,max} = \frac{M_b h_i}{a e r_i}$$

$$= \frac{6000 \times 0.4843d}{\frac{\pi}{4} d^2 \times 0.0157d \times 3.5d}$$

$$= \frac{66763}{d^2}$$

Direct tensile stress:

$$\sigma_d = \frac{P}{a} = \frac{1500}{\frac{\pi}{4} \times d^2} = \frac{1911}{d^2}$$

Total maximum stress, $\sigma_{max} = \sigma_{b\,max} + \sigma_b$

$$= \frac{66763}{d^2} + \frac{1911}{d^2}$$

For design, $\sigma_{max} = \frac{\sigma_{yt}}{\eta}$

$$\frac{68674}{d^2} = \frac{380}{4}$$

$d = 26.89$ mm Say 27mm

result:

Diameter of the Steel bar = 27mm.

2. A simply supported beam has a concentrated load at a centre which fluctuates from a value of P to 4P. the span of the Beam is 500 mm and its cross section is circular with diameter of 60mm, taking for the beam material an ultimate stress of 700 MPa, a yield stress of 500MPa, endurance limit of 330 MPa for reversed bending and a factor for safety of 1.3 calculate the maximum value of P. take a size factor of 0.85 and a surface finish factor of 0.9.

Given data:

$W_{max} = 4P$

$W_{min} = P$

$l = 500$mm

$d = 60$mm

$\sigma_a = 400\,MPa = 700\,N/mm^2$

$\sigma_{-1} = 320\,MPa = 320\,N/mm^2$

$\sigma_y = 500\,MPa = 500\,N/mm^2$

$n = 1.5$

$K_{sz} = 0.85$

$K_{SF} = 0.9.$

Solution:

Maximum bending moment, $M_{b\,max} = \frac{W_{max}}{2} \times \frac{d}{2}$.

$$= \frac{4P}{2} \times \frac{500}{2} = 500P$$

maximum bending stress, $\sigma_{max} = \frac{M_{b\,max}}{Z}$

$$Z = \frac{\pi d^3}{32} = \frac{\pi}{32}(60^3) = 21202.75$$

$$\sigma_{max} = \frac{500P}{2120572} = 0.0235P\ N/mm^2$$

Minimum bending moment, $M_{b\,min} = \frac{W_{min}}{2} \times \frac{d}{2}$

$$= \frac{P}{2} \times \frac{500}{2} = 125P$$

Minimum bending stress, $\sigma_{min} = \frac{M_{b\,min}}{Z}$

$$= \frac{125P}{21202.75} = 0.00589P\ N/mm^2$$

Mean stress $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

$= \frac{0.0235P + 0.00589P}{2}$

$= 0.0147P \ N/mm^2$

Amplitude stress $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$

$= \frac{0.0235P - 0.00589P}{2}$

$\sigma_a = 0.008805 \ P \ N/mm^2$

(i) using Soderberg equation:

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + \frac{k_f \cdot \sigma_a}{\sigma_{-1}(K_L \cdot K_{SF} \cdot K_{SZ})}$$

$$\frac{1}{1.3} = \frac{0.0147P}{500} + \frac{1 \times 0.00880P}{330 \times 0.85 \times 0.}$$

$P = 11967.2N$

using Goodman equation:

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_u} + \frac{k_f \cdot \sigma_a}{\sigma_{-1}(K_L \cdot K_{SF} \cdot K_{SZ})}$$

$$\frac{1}{1.3} = \frac{0.0147P}{700} + \frac{1 \times 0.008805P}{330 \times 0.85 \times 0.9}$$

$P = 13766.2 \ N$

Choosing the lowest value, $P = 11967.2 \ N$

3. A transmision Shaft is supported on two bearings 750 mm apart. Two pulleys C and D are located on the shaft at distances of 100 mm and 300 mm respectively to the left hand bearing. Power is transmitted C to D. The diameted and weight of pulley C are 200 mm and 600 N, and D are 300 mm and 750 N. Ratio of 2 for both pulleys. Power to be 2.5 kW at 300 rpm. The drive C is downwards D is upward angles of 45° to horizontal. The Shaft is made of C45 Steel. using $K_b = 1.5$ and $K_t = 1.2$ design, the shaft.

Given data:

C Diameter $D_c = 200$ mm

D diameter $D_D = 300$ mm

C weight $W_c = 600$ N

D weight $W_D = 750$ N

belt tension, $\frac{T_1}{T_2} = \frac{T_3}{T_4} = 2$

Power $P = 25$ KW $= 25 \times 10^6$ N-mm/s

Speed $N = 300$ rpm.

$K_b = 1.5$ & $K_t = 1.2$ and shaft Material : C45 Steel

To find:

Diameter of the Shaft, d.

Solution:

Torque transmitted.

$$M_t = \frac{P \times 60}{2\pi N} = \frac{25 \times 10^6 \times 60}{2 \times \pi \times 300} = 7.957 \times 10^5 \, N/m$$

Force acting on pulley C:

$$M_t = (T_1 - T_2) R_C = T_1 \left(1 - \frac{T_2}{T_1}\right) R_C$$

$$7.957 \times 10^5 = T_1 \left(1 - \frac{1}{2}\right) 100$$

$$T_1 = 15914 \, N$$

$$T_2 = \frac{T_1}{2} = 7957 \, N$$

$$T_2 = 7957 \, N$$

$$W_C = T_1 + T_2 + W_C = 15914 + 7957 + 600$$

$$W_C = 24471 \, N$$

This load is vertically downwards.

Force acting on Pulley D :

$$M_t = (T_3 - T_4) R_D = T_3 \left(1 - \frac{T_4}{T_3}\right) R_D$$

$$7.957 \times 10^5 = T_3 \left(1 - \frac{1}{2}\right) 150$$

$$\underline{T_3 = 10609.33 \, N}$$

$$\underline{T_4 = 5304.66 \, N}$$

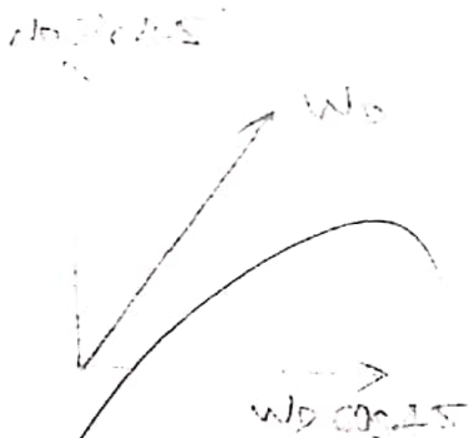
total load on the pulley D.

$$W_D = T_3 + T_4 = 10609.33 + 5304.66$$

$$\underline{W_D = 15994 \, N}$$

Vertical load at D $= (15914 \times \sin 45) + 750$

$$W_{DV} = 12002.9 \, N$$



Horizontal component of W_D

$$W_{DH} = W_D \cos 45'$$

$$= 15914 + \cos 45' = 11252.9 \, N$$

(i) considering vertical loads only:

$$R_{BV} \times 450 = 24471 \times 100 + 12002.9 \times 300$$

$$R_{BV} = 13439N.$$

equalibrium condition.

$$R_{AV} + R_{BV} = 24471 + 12008.9$$
$$= 36473.9N$$

$$R_{AV} = 230349N.$$

(ii) Considering horizontal forces only:

Taking moment about A.

$$R_{BH} \times 450 = 11252.9 \times 300$$

$$R_{BH} = 7501.9N$$

equalibrium condition:

$$R_{AH} + R_{BH} = 11352.9N$$

$$R_{AH} = 11252.9 - 7501.9$$

$$R_{AH} = 3750.9N$$

Bending moment:

Resultant BM = $\sqrt{M_V^2 + M_H^2}$

Resultant BM at C = $\sqrt{(2.3\times10^6)^2 + (0.375\times10^6)^2}$

$= 2.33\times10^6$ N-mm

Resultant BM at D = $\sqrt{(2.01\times10^6)^2 + (1.125\times10^6)^2}$

$= 2.30\times10^6$ N-mm

maximum Value of the BM at C is

$M_b = 2.33\times10^6$ N-mm

equlibrium twissting moment,

$M_{te} = \sqrt{(K_b M_b)^2 + (K_t M_t)^2}$

$= \sqrt{(1.5\times2.33\times10^6)^2 + (1.2\times8\times10^5)^2}$

$M_{te} = 3\times10^6$ N-mm

we know that $M_{te} = \frac{\pi}{16}\times\tau\times d^3$

$3\times10^6 = \frac{\pi}{16}\times47.5\times d^3$

$d = 68.58$ mm

4. Design a muff coupling to connect to shaft transmitting 40kw at 150 rpm. The allowable Stresses key are 37 N/mm² and 96.25 N/mm² respectively. The shear stress is 17.5 N/mm² Assume that the maximum torque is 20% more then the mean torque. The take is width and depth of the parallel key is 22 mm and 14 mm respectively.

Given data:

$P = 40 kW = 40 \times 10^3 W$

$N = 150 rpm$

$\tau_s = 37 N/mm^2$

$\tau_k = 37 N/mm^2$

$\sigma_{cs} = 96.25 N/mm^2$

$\sigma_{ck} = 96.25 N/mm^2$

$\tau_m = 17.5 N/mm^2$

$M_{t\,max} = 1.2 \times M_{t\,mean}$

$b = 22 mm$

$h = 14 mm.$

To find:

Design of muff Coupling.

Solution:

(i) Design of Shaft:

$$M_t\ mean = \frac{P \times 60}{2\pi N} = \frac{40 \times 10^3 \times 60}{2\pi \times 150}$$

$$= 2546.479\ N\text{-}m$$

maximum torque, $M_{t\text{-}max} = 1.2 \times M_t\ mean$

$$= 1.2 \times 2546.479$$

$$M_t\ max = 3055774.9\ N\text{-}mm$$

$$M_t\ max = \frac{\pi}{16} \times \tau_s \times d^3$$

$$3055774.9 = \frac{\pi}{16} \times 37 \times d^3$$

$$d = 74.9\ mm$$

ii) Diameter of the coupling:

a. outer diameter, $D = 2d + 13 = 2 \times 75 + 13$

$$= 163\ mm$$

b. Length of sleeve, $L = 3.5 \times d = 3.5 \times 75$

$$= 262.5\ mm$$

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iii) Design of sleeve:

$$M_{t\,max} = \frac{\pi}{16} \times \tau_s \times \left(\frac{D^4 - d^4}{D}\right)$$

$$305574.9 = \frac{\pi}{16} \times \tau_s \times \left(\frac{(165^4) - (75)^4}{163}\right)$$

$$\tau_s = 3.76\ N/mm^2$$

(iv) Design of key:

(a) check for shear strength:

$$M_t = l \times b \times \tau_k \times \frac{d}{2}$$

$$305574.9 = 131.25 \times 22 \times \tau_k \times \frac{75}{2}$$

$$\tau_k = 28.2\ N/mm^2$$

(b) check for crushing:

$$M_{t\,max} = l \times \frac{h}{2} \times \sigma_c \times \frac{d}{2}$$

$$305574.9 = 131.25 \times \frac{14}{2} \times \sigma_c \times \frac{75}{2}$$

$$\sigma_{ck} = 88.69\ N/mm^2$$

5. A circular shaft 60mm in diameter is welded to a plate. Determine the size of weld if the permissible shear stress in the weld is limited to 85MP

Given data:

$D = 60mm$

$P = 7kN = 7000N$

$\tau = 85 MPa = 85 N/mm$

To find:

Size of weld. h

Solution:

$$\sigma_b = \frac{5.66 M_b}{\pi D^2 h}$$

$M_b = P \times e = 7000 \times 150 = 1050000$ N-mm

$\sigma_b = 2 \times \tau = 2 \times 85 = 170$ N-mm

$$170 = \frac{5.66 \times 1050000}{\pi \times 60^2 \times h}$$

$$h = 3.09 \text{ mm}$$

Area of the weld, $A = t\pi D = 0.707 h\pi D$

$$= 0.707 \times h \times \pi \times 60 = 133h$$

Stress $\tau = \frac{P}{A} = \frac{7000}{133h} = \frac{52.526}{h}$

$$M_b = P \times e = 7000 \times 150 = 1050000 \text{ N-mm}$$

$$Z_w = \frac{\pi t D^2}{4} = \frac{\pi \times 0.707 \times h (60^2)}{4} = 1998.99h$$

$$\sigma_b = \frac{M_b}{Z_w} = \frac{105000}{199899h} = \frac{525.26}{h}$$

Maximum shear stress;

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau^2}$$

$$\frac{1}{2} = \sqrt{\left(\frac{525.26}{h}\right)^2 + 4\left(\frac{52.526}{h}\right)^2}$$

$$85 = \frac{267.83}{h}$$

$$h = 267.83/85$$

hight is $\Rightarrow h = 3.15$ mm

6. A double riveted lap joint is to be made between 6mm plates. The safe working stresses for plates and rivet materials are $\sigma_t = 60\ N/mm^2$, $\sigma_c = 80\ N/mm^2$ $\tau = 50 N/mm^2$

Design the joint.

Given:

Double riveted lap joint

Thickness $t = 6mm$

$\sigma_t = 60\ N/mm^2$

$\sigma_c = 80\ N/mm^2$

$\tau = 50\ N/mm^2$

To find:

Diesign of the Joint.

Solution:

(i) Thickness of the cover plate, t_1:

$t_1 = 0.625 \times t = 0.625 \times 6$

$t_1 = 3.75\ mm.$

(ii) Diameter of the rivet, d.

$$F_s = 2 \times 1 \times \frac{\pi d^2}{4} \times 50$$

$$F_s = 78.53.98 d^2$$

$$F_c = i \times d \times t \times \sigma_c = 2 \times d \times 6 \times 80$$

$$F_c = 960d$$

$$78.5398 d^2 = 960d$$

$$d = 12.22 mm$$

(iii) margin of the rivet, e:

$$e = 1.5d$$
$$= 1.5 \times 13 = 19.5 mm$$

(iv) Distance between two rows, P_b:

$$P_b = 3d$$
$$= 3 \times 13 = 39 mm$$

(V) Pitch of the rivet, P:

$$F_t = (P - d)\, t\, \sigma_t$$
$$= (P - 13) \times 6 \times 60$$
$$F_t = (P - 13) \times 360$$

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$$78.5398 d^2 = (p - 13) \times 360$$

$$78.5398 (13^2) = (p - 13) \times 360$$

$$p = 50mm$$

(vi) Efficiency of the rivet joint, η:

Crushing of the rivet, F_c

$$F_c = 960 \times d = 960 \times 13$$

$$F_c = 12480N$$

$$\eta = \frac{\text{Length of } F_1, F_s \text{ and } F_c}{p \times t \times \sigma_t}$$

$$\eta = \frac{F_c}{p \times t \times \sigma_t}$$

$$= 12480 / 50 \times 6 \times 60$$

$$\eta = 69.33\%$$ of

Efficiency of the rivet joint.

7. A safety valve of 60 mm diameter is to blow off at pressure of 1.2 N/mm^2. It is placed on it seat by a close-coiled helical spring. The maximum lift value 10 mm. Design a suitable spring index 5 and compression of 35 mm. The maximum shear stress is 500 N/mm^2. The spring materials is 0.80×10^5 N/mm^2.

Calculate the

(i) diameter of spring wire

(ii) mean coil diameter

(iii) numbers of active turns

(iv) pitch of the coil.

Given data:

diameter d_v = 60 mm

Pressure, P_v = 1.2 N/mm^2

y_1 = 10 mm

C = 5

y_i = 35 mm

Shear stress τ_{max} = 500 N/mm^2

Modulus G = 0.8×10^5 N/mm^2

To find:

d, D, n, p.

Solution:

(i) Diameter of Spring wire, d:

$$K_s = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times5-1}{4\times5-4} + \frac{0.615}{5}$$

$$K_s = 1.35$$

$$\tau_{max} = K_s \frac{8 P_{max} C}{\pi d^2}$$

$$P_{max} = P \times \frac{y_{max}}{y_i} = 3392.92 \times \frac{45}{35}$$

$$P_{max} = 4362.326 N$$

$$\tau_{max} = \frac{1.35 \times 8 \times 4362.326 \times 5}{\pi d^2}$$

$$d^2 = 145.578$$

$$d = 12.06 mm$$

(ii) mean coil diameter, D

Spring index, $C = \frac{D}{d}$

$D = C \times d$

$D = 5 \times 12.5 = 62.5mm$

$D = 62.5mm$

(iii) Number of active turns, n:

$y_{max} = \frac{8 P_{max} C^3 n}{G d}$

$45 = \frac{8 \times 4362326 \times 5^3 \times n}{0.8 \times 10^5 \times 12.5}$

$n = 10.48$ Coils.

total numbers of coils: $n_t = n + 3 = 10.48 + 3$

$n = 13.5$ coils.

(iv) Pitch of the coil, p

$p = \frac{L_f - L_s}{n_t} + d$

Solid length $L_s = dn + 3d = 12.5 \times 10.5 + 3 \times 12.5 = 168.75mm$

Free Length $L_f = L_s + y_i = 168.75 + 35 = 203.75mm$

Pitch, $p = \frac{203.75 - 168.75}{13.5} + 12.5 = 15.09mm$

$P = 15.09mm$

8. A single Cylinder double acting cylinder steam delivers 185 kW at 100 rpm. The maximum fluctuation of energy per revolution is 15% of energy developed per revolution. The speed variation is limited to 1% either way from the mean. The diameter of the rim is 2.4 m.

Given:

power $= 185\,kW = 185 \times 10^3\,W$

$N = 100\,rpm$

energy, $\Delta E = 15\%$ developed E.

Mean diameter of the rim $D = 2.4\,m$

$\therefore R = 1.2\,m.$

Solution:

1. Mass of the flywheel rim:

$$E = \frac{P \times 60}{N} = \frac{185 \times 10^3 \times 60}{100} = 111000\,N\text{-}m$$

Maximum fluctuation of energy:

$$\Delta E = 0.15\,E = 0.15 \times 111000$$

$$\Delta E = 16650\,N\text{-}m$$

$K_s = \frac{N_1 - N_2}{N_1} = 0.02$

Velocity of the flywheel

$V = \frac{\pi D N}{60} = \frac{\pi \times 2.4 \times 100}{60} = 12.57\ m/s$

Fluction energy (ΔE)

$16650 = m V^2 K_s$

$= m(12.57)^2\ 0.02$

$m = 5268.99\ kg$

2. Cross-sectional diameters of the flywheel rim:

Cross-sectional are of rim,

$A = b \times h = 2h \times h = 2h^2$

Flywheel rim (m)

$5268.99 = A \times \pi D \times \rho = 2h^2 \times \pi \times 2.4 \times 7200$

$[\rho = 7200\ kg/m^3]$

$5298.99 = 108588 h^2$

$h^2 = 5270 / 108588$

$h = 0.22 m = 220 mm$

$b = 2h = 2 \times 220 = 440 mm$

we also know that maximum torque transmitted by the shaft

$$35.33 \times 10^6 = \frac{\pi}{16} \times \tau \times d_1^3$$

$$= \frac{\pi}{16} \times 40 \times d_1^3$$

$$d_1 = 165 mm$$

$d = 2d_1 = 2 \times 165 = 330$

and $l = b = 440 mm$

4. Cross-sectional dimension of the elliptical arms:

a: Major axis

c: Minor axis $[c = 0.5a]$

n: Numbers of arms $[n = 6)$.

$\sigma_b = 14$ MPa

$\sigma_b = 14 N/mm^2$

$$M = \frac{(M_t)}{D \cdot n} (D - d)$$

$$= \frac{176662 \times 10^3}{2.4 \times 6} (2.4 \times 0.33) = 2599.5 \times 10^3$$

$$Z = \frac{\pi}{32} c a^2 = \frac{\pi}{32} (0.5a) a^2 = 0.05 a^3$$

Stress (σ_{fa})

$$14 = \frac{M}{Z}$$

$$14 = \frac{10 + 560 \times 10^3}{a^3}$$

major axis: $a = 155$ mm

$$C = 0.05a$$

$$C = 0.05 \times 155$$

$$C = 78 \text{ mm}$$

5. Dimensions of key:

Width of key $w = 45$ mm

thickness, $t = 25$ mm

$$35.33 \times 10^6 = L_k \times w \times \tau \times \frac{d_1}{2}$$

$$35.33 \times 10^6 = L_k \times 45 \times 40 \times \frac{165}{2}$$

$L_k = 238$ mm

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9. A full journal bearings of 100mm diameter and 150 long supports a radial load of 6KN. The shaft rotates at 560rpm. The diametral clearance is 0.15 mm. The room temperature is 25°c, and the operating temperature is 70°C. The bearing is well ventilated and so no artificial cooling is required. Suggest a suitable oil to meet the requirement.

Given

$D = 100mm = 10cm$

$L = 150mm = 15cm$

$W = 6KW = 6000N = 611.62 \, kgf$

$n = 560 \, rpm$

$C = 0.15mm = 0.015cm$

$T_a = 25°C$

$T = 70°C$

Solution:

Velocity $V = \frac{\pi D N}{60} = \frac{\pi \times 0.1 \times 560}{60}$

$V = 176 \, m/min$

Heat dissipated $H_d = \frac{((\Delta t)+18)^2 LD}{K}$

$\Delta t = 70 - 25 = 45°c$

From PSGDB, $K = 437$.

$$H_d = \frac{(45 \times 18)^2 \times 10 \times 15}{437} = 1362.36 \frac{kgf\text{-}m}{min}$$

Cooling is required:

$$H_d = H_g$$
$$= \mu W V$$
$$= \mu \times 611.62 \times 176$$
$$1362.36 = 107601.8 \mu$$
$$\mu = 0.01266$$
$$\frac{L}{D} = 1.5$$

From chart on PSGDB for $\frac{L}{D} = 1.5$ $K \simeq 0.00$

$$\mu = \frac{33.25}{10^{10}} \left(\frac{Zn}{P}\right)\left(\frac{D}{C}\right) + K$$

$$P = \frac{W}{LD} = \frac{611.62}{10 \times 15} = 4077 \, kgf/cm^2$$

$$D/C = \frac{15}{0.015} = 1000$$

$$\therefore 0.01266 = \frac{33.25}{10^{10}}\left(\frac{2\times 560}{4.077}\right)(1000)+0.0021$$

$$Z = 22.246 \text{ cP}, \approx 30 \text{ cP}$$

for 70°C and 30cP, the oil to be used is SAE 40.

10. Determine the dynamic load carrying capacity of a deep-groove ball bearing with the least bore size and which is required to resist a radial load of 4kN, and an axial thrust load of 3kN. The shaft rotates at 1400 rpm. The bearing is required to be in an operation for 12000 hours, with 90% reliability.

Given:

$F_r = 4\text{kN} = 4000\text{N}$

$F_a = 3\text{kN} = 3000\text{N}$

$N = 1400 \text{ rpm}$

90% reliability

$L_h = 1200 \text{ hrs}.$

Solution:

$$\frac{F_a}{F_r} = \frac{3000}{4000} = 0.75$$

e = 0.44 since $\frac{F_a}{F_r} > e$, we choose

$X = 0.56$ and $Y = 1$

Assuming a service factor, $S = 1.2$,

$$P = (XF_r + YF_a)S$$

$$P = [(0.56 \times 4000) + (1 \times 3000)] \times 1.2$$

$$P = 10480\ N$$

At 1400 rpm, for 12000 hours, from Chart on PSGDB 4.6 we have approximately,

$$\frac{C}{P} = 10.60$$

$$C = 10.60 \times P = 10.6 \times 10480$$

$$C = 111088\ N$$

It means that the [illegible] [illegible] capacity should be equal to or more than [illegible]. After the [illegible] [illegible] should be the least [illegible]

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Assignment
Sample paper
[2022-2023]
EVEN SEMESTER

ASSIGNMENT

Reg no: 912020114001

NAME:- AJAY GOWTHAM

Sub code :- ME8692

Sub Name :- Finite element Analysis.

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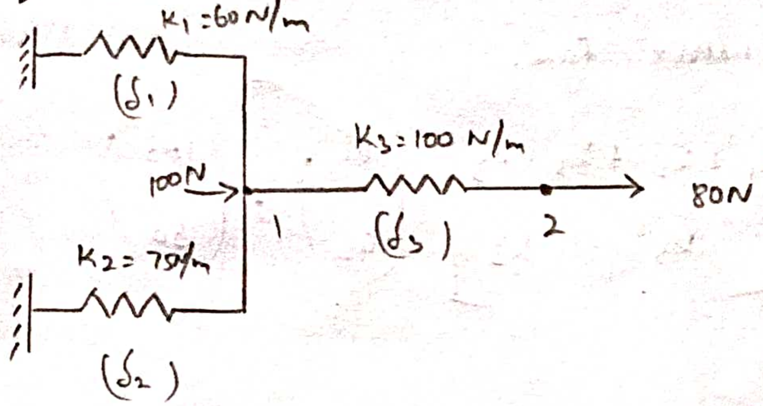
2.) Determine the displacements of nodes 1 and 2 in the spring system shown in figure use minimum of Potential energy principle to assemble equations of equilibrium



Given data:

$k_1 = 60$ N/m. $\quad F_1 = 100$ N

$k_2 = 75$ N/m $\quad F_2 = 80$ N

$k_3 = 100$ N/m

To find: Displacements of nodes 1 and 2

Soln:

$\delta_1 = u_1, \quad \delta_2 = u_1, \quad \delta_3 = u_2 - u_1$

π = Strain energy − work done

$$\pi = \frac{1}{2} k_1 \delta_1^2 + \frac{1}{2} k_2 \delta_2^2 + \frac{1}{2} k_3 \delta_3^2 - 100u_1 - 80u_2$$

$$\pi = \frac{1}{2} k_1 u_1^2 + \frac{1}{2} k_2 u_1^2 + \frac{1}{2} k_3 (u_2 - u_1)^2 - 100u_1 - 80u_2 \quad \rightarrow (1)$$

Now $\frac{\partial \pi}{\partial u_1} = 0$

$$k_1 u_1 + k_2 u_2 + k_3 (u_2 - u_2)(-1) - 100 = 0$$

$$(k_1 + k_2 + k_3) u_1 - k_3 u_2 = 100 \quad \rightarrow (2)$$

$\frac{\partial x}{\partial u_2} = 0 \rightarrow \quad k_3(u_2 - u_1) - 80 = 0$

$-k_3 u_1 + k_3 u_2 = 80 \quad \rightarrow (3)$

(2) & (3) matrix form.

$$\begin{bmatrix} k_1 + k_2 + k_3 & -k_3 \\ k_3 & k_3 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 100 \\ 80 \end{pmatrix}$$

Values of k_1, k_2 and k_3

$$\begin{Bmatrix} 60 + 75 + 100 & -100 \\ -100 & 100 \end{Bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{pmatrix} 100 \\ 80 \end{pmatrix}$$

$$\begin{pmatrix} 235 & -100 \\ -100 & 100 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 100 \\ 80 \end{pmatrix}$$

$235 u_1 - 100 u_2 = 100$

$-100 u_1 + 100 u_2 = 80$

$135 u_1 = 180$

$u_1 = \frac{180}{135}$

$u_1 = 1.333$

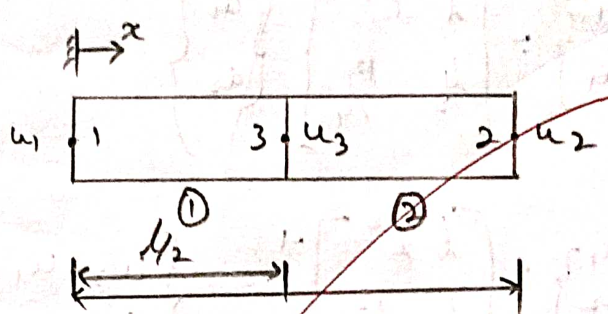
u_1 - Value in equation 5

$-100(1.333) + 100 u_2 = 80$

$u_2 = 2.133$

3) Derivation of shape function for one dimensional quadratic bar element.

Consider a quadratic bar element with nodes 1, 2 and 3 as shown in fig u_1-u_2 and u_3 are the displacements at the respective nodes so, u_1-u_2 and u_3 are considered as degrees of freedom of this quadratic bar element



$$u = a_0 + a_1 x + a_2 x^2$$

$$u = [1 \; x \; x^2] \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$u = u_1, \; x = 0$

$u = u_2, \; x = l$

$u = u_3, \; x = l/2$

$u_1 = a_0$

$u_2 = a_0 + a_1 l + a_2 l^2$

$u_3 = a_0 + a_1 (l/2) + a_2 (l/2)^2$

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$$u_2 = u_1 + a_1 l + a_2 l^2$$

$$u_3 = u_1 + \frac{a_1 l}{2} + \frac{a_2 l^2}{4}$$

$$u_2 - u_1 = a_1 l + a_2 l^2$$

$$u_3 - u_1 = \frac{a_1 l}{2} + \frac{a_2 l^2}{4}$$

$$\begin{Bmatrix} u_2 - u_1 \\ u_3 - u_1 \end{Bmatrix} = \begin{bmatrix} l & l^2 \\ l/2 & l^2/4 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{bmatrix} l & l^2 \\ l/2 & l^2/4 \end{bmatrix}^{-1} \begin{Bmatrix} u_2 - u_1 \\ u_3 - u_1 \end{Bmatrix}$$

$$= \frac{1}{\left(\frac{l^3}{4} - \frac{l^3}{2}\right)} \begin{bmatrix} \frac{l^2}{4} & -l^2 \\ \frac{-l}{2} & l \end{bmatrix} \begin{Bmatrix} u_2 - u_1 \\ u_3 - u_1 \end{Bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{(a_{11} a_{22} - a_{12} a_{21})} \times \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$$\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \frac{1}{\left(\frac{-l^3}{4}\right)} \begin{bmatrix} \frac{l^2}{4} & -l^2 \\ -l/2 & l \end{bmatrix} \begin{Bmatrix} u_2 - u_1 \\ u_3 - u_1 \end{Bmatrix}$$

$$a_1 = \frac{-4}{l^3}\left[\frac{l^2}{4}(u_2-u_1) - l^2(u_3-u_1)\right]$$

$$a_2 = \frac{-4}{l^3}\left[-\frac{l}{2}(u_2-u_1) + l(u_3-u_1)\right]$$

$$a_1 = \frac{-4}{l^3}\cdot\left[\frac{l^2u_2}{4} - \frac{l^2u_1}{4} - l^2u_3 + l^2u_1\right]$$

$$= \frac{-4l^2u_2}{4l^3} + \frac{4l^2u_1}{4l^3} + \frac{4l^2u_3}{l^3} - \frac{4l^2u_1}{l^3}$$

$$= \frac{-u_2}{l} + \frac{u_1}{l} + \frac{4u_3}{l} - \frac{4u_1}{l}$$

$$a_1 = \frac{-3u_1}{l} - \frac{u_2}{l} + \frac{4u_3}{l}$$

$$a_2 = \frac{-4}{l^3}\left[\frac{-lu_2}{2} + \frac{1}{2}u_1 + lu_3 - lu_1\right]$$

$$= \frac{4lu_2}{2l^3} - \frac{4l}{2l^3}u_1 - \frac{4l}{l^3}u_3 + \frac{4l}{l^3}u_1$$

$$= \frac{2u_2}{l^2} - \frac{2}{l^2}u_1 - \frac{4}{l^2}u_3 + \frac{4}{l^2}u_1$$

$$a_2 = \frac{2}{l^2}u_1 + \frac{2u_2}{l^2} - \frac{4}{l^2}u_3$$

$$\begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-3}{l} & -\frac{1}{l} & \frac{4}{l} \\ \frac{2}{l^2} & \frac{2}{l^2} & \frac{-4}{l^2} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$(u) = \{1 \ x \ x^2\} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{l} & -\frac{1}{l} & \frac{4}{l} \\ \frac{2}{l^2} & \frac{2}{l^2} & \frac{-4}{l^2} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$(u) = \left[\left(1 - \frac{3}{l}x + \frac{2x^2}{l^2}\right) \left(\frac{-x}{l} + \frac{2x^2}{l^2}\right) \left(\frac{4x}{l} - \frac{4x^2}{l^2}\right)\right] \{$$

$$u = [N_1 \ N_2 \ N_3] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$u = N_1 u_1 + N_2 u_2 + N_3 u_3$$

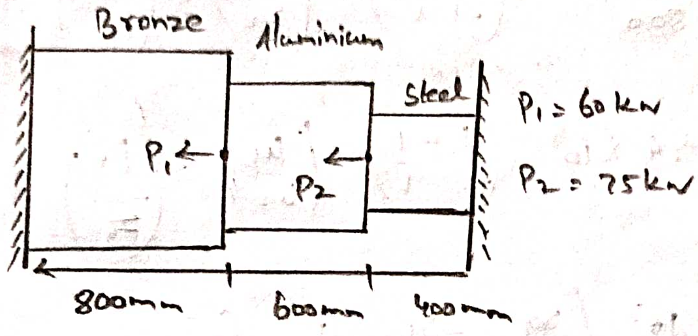
$$N_1 = 1 - \frac{3x}{l} + \frac{2x^2}{l^2}$$

$$N_2 = \frac{-x}{l} + \frac{2x^2}{l^2}$$

$$N_3 = \frac{4x}{l} - \frac{4x^2}{l^2}$$

4.) The composite Structure shown in fig is Subjected to a bar element. Determine the displacements, stresses and support reactions

Bronze: $A = 2400mm^2$, $E = 38GPa$ Aluminum: $A = 1200mm^2$, $E = 70GPa$, Steel $A = 600mm^2$, $E = 200GPa$



Given data:

$P_1 = 60kN = -60 \times 10^3 N$

$P_2 = 75kN = -75 \times 10^3 N$

$l_1 = 800mm$, $l_2 = 600mm$, $l_3 = 400mm$

$A_1 = 2400mm^2$

$E_1 = 83 GPa = 83 \times 10^9 Pa$

$= 83 \times 10^9 N/m^2 = 83 \times 10^3 N/mm^2$

$A_2 = 1200mm^2$

$E_2 = 70 GPa = 70 \times 10^3 N/mm^2$

$A_3 = 600mm^2$

$E_3 = 200 GPa = 200 \times 10^3 N/mm^2$

$a_3 = 11.7 \times 10^{-6}/°C$

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$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\frac{A_2 E_2}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$\frac{2400 \times 83 \times 10^3}{800} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$$

$$249 \times 10^3 \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$$

$$10^3 \begin{pmatrix} 249 & -249 \\ 249 & 249 \end{pmatrix}_{2}^{1} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$$

$$\frac{A_2 E_2}{l_2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{pmatrix} u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} F_2 \\ F_3 \end{pmatrix}$$

$$\frac{1200 \times 7 \times 10^3}{600} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} F_2 \\ F_3 \end{pmatrix}$$

$$10^3 \begin{pmatrix} 140 & -140 \\ -140 & 140 \end{pmatrix}_{3}^{2} \begin{pmatrix} u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} F_2 \\ F_3 \end{pmatrix}$$

$$\frac{A_3 E_3}{l_3} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{pmatrix} F_3 \\ F_4 \end{pmatrix}$$

$$\frac{600 \times 200 \times 10^3}{400} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix}$$

$$10^2 \begin{bmatrix} 300 & -300 \\ -300 & 300 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \begin{pmatrix} F_2 \\ F_4 \end{pmatrix}$$

$$10^3 \begin{bmatrix} 249 & -249 & 0 & 0 \\ -249 & 249+140 & -140 & 0 \\ 0 & -140 & 140+300 & -300 \\ 0 & 0 & -300 & 300 \end{bmatrix} \times \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

$$10^3 \begin{bmatrix} 249 & -249 & 0 & 0 \\ -249 & 389 & -140 & 0 \\ 0 & -140 & 440 & -300 \\ 0 & 0 & -330 & 300 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

$$10^3 \begin{bmatrix} 249 & -249 & 0 & 0 \\ -249 & 389 & -140 & 0 \\ 0 & -140 & 440 & -300 \\ 0 & 0 & -300 & 300 \end{bmatrix} \begin{pmatrix} 0 \\ u_2 \\ u_3 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0 \\ -60 \times 10^3 \\ -75 \times 10^3 \\ 0 \end{Bmatrix}$$

$$\Rightarrow 10^3 \begin{bmatrix} 389 & -140 \\ -140 & 440 \end{bmatrix} \begin{pmatrix} u_2 \\ u_3 \end{pmatrix} = \begin{Bmatrix} -60 \times 10^3 \\ -75 \times 10^3 \end{Bmatrix}$$

$u_2 = 0.243$ mm

$u_3 = -0.247$ mm

$$\sigma_1 = \frac{E_1 (u_2 - u_1)}{L_1}$$

$$\sigma_1 = \frac{83 \times 10^3 (-0.243)}{800}$$

$$\sigma_1 = -25.21 \ N/mm^2$$

$$\sigma_2 = \frac{70 \times 10^3 \times (-0.247 + 0.243)}{600}$$

$$\sigma_2 = -4.66 \ N/mm^2$$

$$\sigma_3 = \frac{E_3 (u_4 - u_3)}{L_3}$$

$$= \frac{200 \times 10^3 (0 + 0.247)}{400}$$

$$\sigma_3 = 123.5 \ N/mm^2$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{Bmatrix} = 10^3 \begin{bmatrix} 244 & -244 & 0 & 0 \\ -244 & 384 & -140 & 0 \\ 0 & -140 & 400 & -300 \\ 0 & 0 & -300 & 300 \end{bmatrix} \begin{Bmatrix} 0 \\ -0.243 \\ -0.247 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ -60 \times 10^3 \\ -75 \times 10^3 \\ 0 \end{Bmatrix}$$

$$R_1 = 6.05 \times 10^4 N$$

$$R_2 = 10^3 \left[(389 \times -0.243) + (-140 \times -0.247) \right] - (-60 \times 10^3)$$

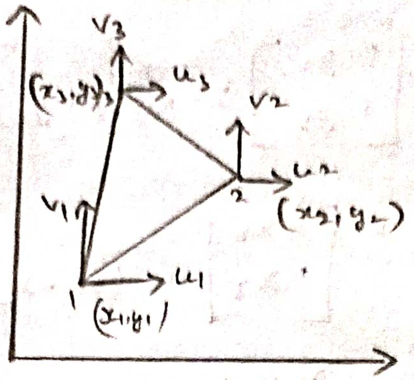
$$R_2 = 0 N$$

$$R_3 = 0 N$$

$$R_4 = 10^3 (-300 \times -0.247)$$

$$R_4 = 7.4 \times 10^4 N$$

5) we consider this CST element because its derivation is the simplest among the available two dimensional elements.



consider a typical CST element with nodes 1, 2 and 3 as shown in fig let the nodal displacements be u_1, u_2 u_3 v_1 v_2 and v_3

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y \\ 1 & x_3 & y_2 \end{bmatrix}^{-1} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$D = \begin{bmatrix} + & - & + \\ 1 & x_1 & y_1 \\ - & + & - \\ 1 & x_2 & y_2 \\ + & - & + \\ 1 & x_2 & y_3 \end{bmatrix}$$

$$D^{-1} = \frac{C^T}{D}$$

$C_{11} = + \begin{bmatrix} x_2 & y_2 \\ x_3 & y_3 \end{bmatrix} = (x_2 y_3 - x_3 y_2)$

$C_{12} = - \begin{bmatrix} 1 & y_2 \\ 1 & y_3 \end{bmatrix} = -(y_3 - y_2) = y_2 - y_3$

$C_{13} = + \begin{bmatrix} 1 & x_2 \\ 1 & x_3 \end{bmatrix} = (x_3 - x_2)$

$C_{21} = \begin{bmatrix} x_1 & y_1 \\ x_3 & y_3 \end{bmatrix} = -(x_1 y_3 - x_3 y_3) = x_3 y_1 - x_1 y_3$

$C_{22} = + \begin{bmatrix} 1 & y_1 \\ 1 & y_3 \end{bmatrix} = y_3 - y_1$

$C_{23} = \begin{bmatrix} 1 & x_1 \\ 1 & x_3 \end{bmatrix} = -(x_3 - x_1) = x_1 - x_3$

$C_{31} = + \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} = x_1 y_2 - x_2 y_1$

$C_{32} = - \begin{bmatrix} 1 & y_1 \\ 1 & y_2 \end{bmatrix} = -(y_2 - y_1) = y_1 - y_2$

$C_{33} = + \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix} = x_2 - x_1$

$$C_T = \begin{bmatrix} (x_2 y_3 - x_3 y_2) & (x_3 y_1 - x_1 y_3) & (x_1 y_2 - x_2 y_1) \\ (y_2 - y_3) & (y_3 - y_1) & (y_1 - y_2) \\ (x_3 - x_2) & (x_1 - x_3) & (x_2 - x_1) \end{bmatrix}$$

$$D = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

$|D| = 1(x_2 y_3 - x_3 y_2) - x_1(y_3 - y_2)$

$$\begin{bmatrix} (x_2 y_3 - x_3 y_2) & (x_3 y_1 - x_1 y_3) & (x_1 y_2 - x_2 y_3) \\ y_2 - y_3 & y_3 - y_1 & y_1 - y_2 \\ x_3 - x_2 & x_1 - x_2 & x_2 - x_1 \end{bmatrix}$$

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}^{-1} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} (x_2 y_3 - x_3 y_2) & (x_3 y_1 - x_1 y_2) & (x_1 y_2 - x_2 y_3) \\ y_2 - y_3 & y_3 - y_1 & y_1 - y_2 \\ x_3 - x_2 & x_1 - x_3 & x_2 - x_1 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

$$A = \frac{1}{2} \quad \frac{1}{2} \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$$

$$2A = (x_2 y_3 - x_3 y_3) - x_1 (y_3 - y_2) + (y_1 (x_3 - x_2))$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \frac{1}{2A} \begin{bmatrix} (x_2 y_3 - x_3 y_2) & (x_3 y_1 - x_1 y_3) & (x_1 y_2 - x_2 y_3) \\ y_2 - y_3 & y_3 - y_1 & y_1 - y_2 \\ x_3 - x_2 & x_1 - x_2 & x_2 - x_1 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

$p_1 = x_2 y_3 - x_3 y_2$ $\quad$ $p_2 = x_3 y_1 - x_1 y_3$ $\quad$ $p_3 = x_1 y_2 - x_2 y_1$

$q_1 = y_2 - y_3$ $\quad$ $q_2 = y_3 - y_1$ $\quad$ $q_3 = y_1 - y_2$

$r_1 = x_3 - x_2$ $\quad$ $r_1 = x_1 - x_3$ $\quad$ $r_3 = x_2 - x_1$

$$u = a_1 + a_2 x + a_3 y$$

$$u = (1\ x\ y)\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$u = [1\ x\ y] \times \frac{1}{2A} \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$= \frac{1}{2A} [1\ x\ y] \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

$$= u = \begin{bmatrix} \frac{p_1 + q_1 x + r_1 y}{2A} & \frac{p_2 + q_2 x + r_2 y}{2A} & \frac{p_3 + q_3 + r_3 y}{2A} \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

$$u = (N_1\ N_2\ N_3) \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

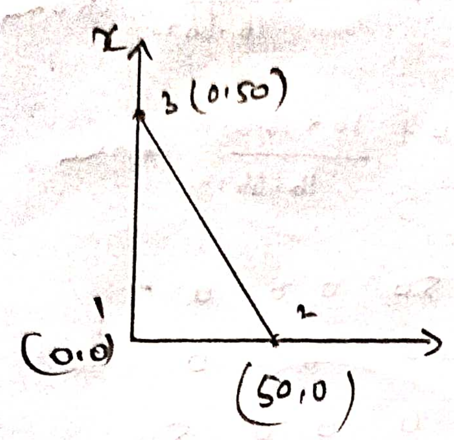
$$N_1 = \frac{p_1 + q_1 x + r_1 y}{2A}$$

$$N_2 = \frac{p_2 + q_2 x + r_2 y}{2A}$$

$$N_3 = \frac{p_3 + q_3 x + r_3 y}{2A}$$

$$u = \begin{Bmatrix} u(x,y) \\ v(x,y) \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \times \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix}$$

7.) For the Axi's Symmetric elements shown in fig determine the stiffness matrix let $E = 2.1 \times 10^5 \, N/mm^2$ and $\nu = 0.25$, the coordinates shown in figure are in millimeters



$K = 2\pi r A [B]^T [D] [B]$

$A = \frac{1}{2} \times 50 \times 50$

$A = 1250 mm^2$

$r = \frac{0+50+0}{3}$

$r = 16.667 mm$

$z = \frac{z_1 + z_2 + z_3}{3} = \frac{0+0+50}{3}$

$z = 16.667 mm$

$$D = \frac{2.1 \times 10^5}{(1+0.25)(1-(2 \times 0.25))} \begin{bmatrix} 1-0.25 & 0.25 & 0.25 & 0 \\ 0.25 & 1-0.25 & 0.25 & 0 \\ 0.25 & 0.25 & 1-0.25 & 0 \\ 0 & 0 & 0 & \frac{1-(2\times0.25)}{2} \end{bmatrix}$$

$$D = 84 \times 10^3 \begin{bmatrix} 3 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\alpha_1}{\gamma} + \beta_1 + \frac{\gamma_1 x}{\gamma} = \frac{2500}{16.667} + (-50) + \frac{-50 \times 16.667}{16.667} = 50\,mm$$

$$\frac{\alpha_2}{\gamma} + \beta_2 + \frac{\gamma_2 x}{\gamma} = \frac{0}{16.667} + 50 + \frac{0 \times 16.667}{16.667} = 50\,mm$$

$$\frac{\alpha_3}{\gamma} + \beta_3 + \frac{\gamma_3 x}{\gamma} = \frac{0}{16.667} + 0 + \frac{50 \times 16.667}{16.667} = 50\,mm$$

$$B = \frac{1}{2 \times 1250} \begin{bmatrix} -50 & 0 & 50 & 0 & 0 & 0 \\ 50 & 0 & 50 & 0 & 50 & 0 \\ 0 & -50 & 0 & 0 & 0 & 50 \\ -50 & -50 & 0 & 50 & 50 & 0 \end{bmatrix}$$

$$B = 0.02 \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$(D)(B) = 1680 \begin{bmatrix} -2 & -1 & 4 & 0 & 1 & 1 \\ 2 & -1 & 4 & 0 & 3 & 1 \\ 0 & -3 & 2 & 0 & 1 & 3 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$B^T D B = 0.002 \begin{bmatrix} -1 & 1 & 0 & -1 \\ 0 & 0 & -1 & -1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times 1680 \begin{bmatrix} -2 & -1 & 4 & 0 \\ 2 & -1 & 4 & 0 & 3 \\ 0 & -3 & 2 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 \end{bmatrix}$$

$$= 33.6 \begin{bmatrix} 5 & 1 & 0 & -1 & 1 & 0 \\ 1 & 4 & -2 & -1 & -2 & -3 \\ 0 & -2 & 8 & 0 & 4 & 2 \\ -1 & -1 & 0 & 1 & 1 & 0 \\ 1 & -2 & 4 & 1 & 4 & 1 \end{bmatrix}$$

$$K = 2\pi r A \times 33.6 \begin{bmatrix} 5 & 1 & 0 & -1 & 1 & 0 \\ 1 & 4 & -2 & -1 & -2 & -3 \\ 0 & -2 & 8 & 0 & 4 & 2 \\ -1 & -1 & 0 & 1 & 1 & 0 \\ 1 & -2 & 4 & 1 & 4 & 1 \\ 0 & -3 & 2 & 0 & 1 & 3 \end{bmatrix}$$

$$K = 4.34 \times 10^{6} \begin{bmatrix} 5 & 1 & 0 & -1 & 1 & 0 \\ 1 & 4 & -2 & -1 & -2 & -3 \\ 0 & -2 & 8 & 0 & 4 & 2 \\ -1 & -1 & 0 & 1 & 1 & 0 \\ 1 & -2 & 4 & 1 & 1 & 0 \\ \text{~~1~~} & \text{~~-2~~} & \text{~~4~~} & \text{~~1~~} & \text{~~4~~} & \text{~~1~~} \\ 0 & -3 & 2 & 0 & 1 & 3 \end{bmatrix}$$

$$K = 4.34 \times 10^{6} \begin{bmatrix} 5 & 1 & 0 & -1 & 1 & 0 \\ 1 & 4 & -2 & -1 & -2 & -3 \\ 0 & -2 & 8 & 0 & 4 & 2 \\ -3 & -1 & 0 & 1 & 1 & 0 \\ 1 & -2 & 4 & 1 & 4 & 1 \\ 0 & -3 & 2 & 0 & 1 & 3 \end{bmatrix}$$

1.) General steps of the finite element analysis

This section present the general procedure of finite element analysis. For simplicity's sake we will consider only the structural problems.

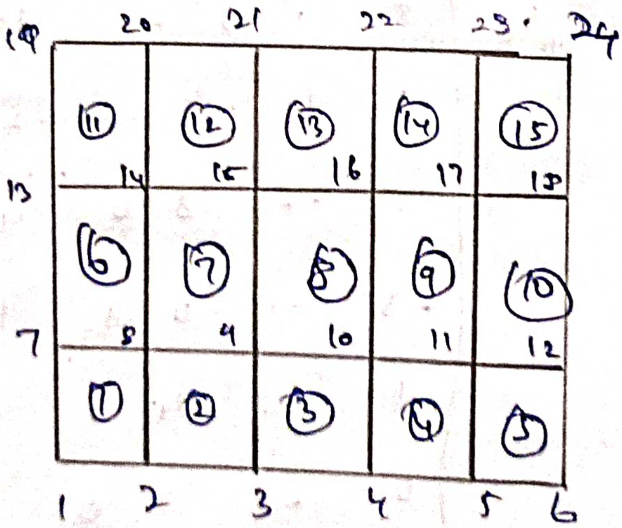
i.) Force method

ii.) Displacement or stiffness method.

Numbering of Nodes and Elements

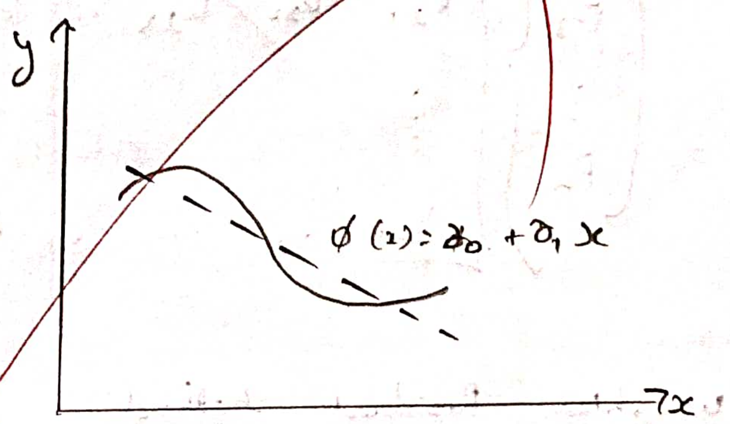
The nodes and elements should be numbered after discretization process.

$$\begin{Bmatrix} \text{Maximum} \\ \text{node number} \end{Bmatrix} - \begin{Bmatrix} \text{Minimum} \\ \text{node number} \end{Bmatrix} = \text{Minimum}$$



Selection of a Displacement Function or Interpolation Function

It involves choosing a displacement function within each element polynomial of linear, quadratic and cubic form are frequently used as displacement functions because they are simple to work within finite element formulation.



It is easy to formulate and computerize the finite element equations.

Define the material behaviour by using Strain - Displacement and Stress Strain Relationships.

Strain - Displacement and Stress Strain relationships are necessary for obtaining the equations for each finite element.

$$e = \frac{du}{dx}$$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ F_n \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & \cdots & k_{1x} \\ k_{21} & k_{22} & k_{23} & \cdots & k_{2x} \\ \vdots & & & & \\ k_{n1} & \cdots & \cdots & \cdots & k_{nx} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_n \end{Bmatrix}$$

Weighted Residual method

This method is Galerkin's method, useful for developing the element equations in thermal analysis problems. They are especially useful when a functional such as potential energy is not readily available.

$$F = [K](u)$$

F = Global force vector

K = Global stiffness matrix

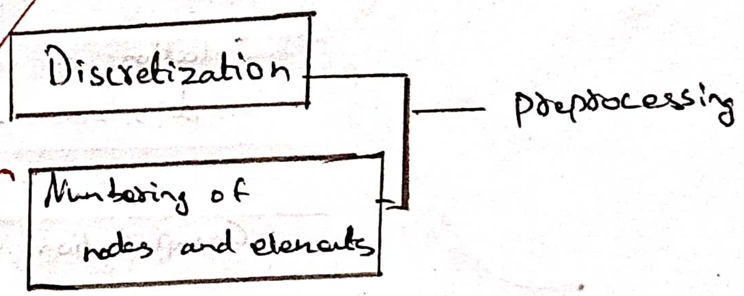
u = Global displacement vector.

Applying boundary conditions:

From equation (1.5) we know that global stiffness matrix [K] is a singular matrix because its determinant is equal to zero.

Solution for the unknown displacements:

A set of simultaneous algebraic equations formed in step 6 can be written in expanded matrix form as follows.



Selection of displacement function

↓

Define the material behaviour

↓

Derivation of element stiffness matrix and equations

↓

Assemble the element equations

↓

Applying boundary conditions

↓

Solution of unknown displacements

↓

Computation of the element stresses and strains.

↓

Interpret the results

9. Shape function for 4 noded rectangular parent element by using natural coordinate system and coordinate transformation (Two dimensional)

Co-ordinates $\varepsilon = -1$, $\eta = -1$

$N_1 =$ 1 at node 1

$N_1 =$ 0 at nodes 2, 3, and 4

$N_1 = c(1-\varepsilon)(1-\eta)$

$N_2 = c(1+1)(1+1)$

$N_1 = 4c$

$c = 1/4$

$N_1 = 1/4 (1-\varepsilon)(1-\eta)$

$N_2 = c(1+1)(1+1)$

$N_2 = 4c$

$1 = 4c$

$c = 1/4$

$N_2 = 1/4 (1+\varepsilon)(1-\eta)$

$N_3 = c(1+1)(1+1)$

$N_3 = 4c$

$1 = 4c$

$c = 1/4$

$N_3 = \frac{1}{4}(1+\varepsilon)(1+\eta)$

$N_4 = c(1+1)(1+1)$

$N_4 = 4c$

$1 = 4c$

$c = \frac{1}{4}$

$N_4 = \frac{1}{4}(1-\varepsilon)(1+\eta)$

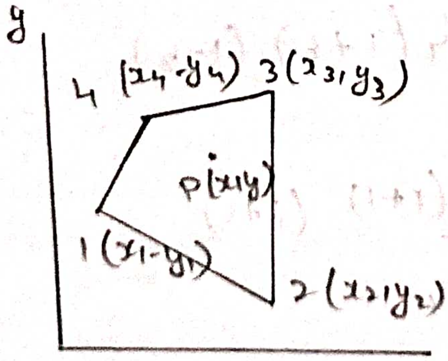
$u = N_1 u_1 + N_2 u_2 + N_3 u_3 + N_4 u_3$

$V = N_1 v_1 + N_2 v_2 + N_3 v_3 + N_4 v_4$

$$u = \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ v_3 \\ u_3 \\ u_4 \\ v_4 \end{bmatrix}$$

$x = N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4$

$y = N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4$



$$u = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{bmatrix}$$

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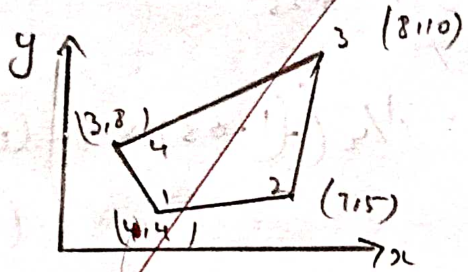
10) Evaluate the Jacobian matrix at the local coordinates $\xi = \eta = 0.5$ for the isoparametric quadrilateral element with its global coordinates as shown in fig. Also evaluate the strain displacement matrix



$x_1 = 4$ $\quad y_1 = 4$

$x_2 = 7$ $\quad y_2 = 5$

$x_3 = 8$ $\quad y_3 = 10$

$x_4 = 3$ $\quad y_4 = 8$

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

$J_{11} = 1/4\ (-(1-b)x_1 + (1-\eta)x_2 + (1+\eta)x_3 - (1+$

$J_{11} = \frac{1}{4}[-(1-0.5)4 + (1-0.5)7 + (1+0.5)8 - ($

$J_{11} = 2.25$

$J_{12} = 1/4[-(1-0.5)\times 4 + (1-0.5)\times 5 + (1+0.5$

$- (1+0.5)8]$

$J_{12} = 0.875$

$J_{21} = 1/4(-(1-0.5)4 - (1+0.5)7 +$

$(1+0.5)8 + (1-0.5)$

$J_{21} = 0.25$

$J_{22} = 1/4(-(1-0.5)\times 4 - (1+0.5)\times 5 +$

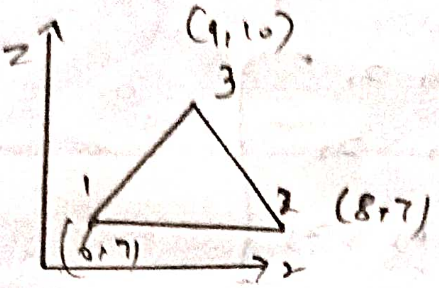
$(1+0.5)10 + (1-0$

$J_{22} = 2.375$

$= \begin{matrix} 2.25 & 0.875 \\ 0.25 & 2.375 \end{matrix}$

$$J = \begin{bmatrix} 2.25 & 0.875 \\ 0.25 & 2.375 \end{bmatrix}$$

8. Calculate the element stiffness matrix and the thermal force vector for the axisymmetric triangular element shown in Fig



$[k] = 2\pi r A [B]^T [D] [B]$

$$B = \frac{1}{2A}\begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 \end{bmatrix}$$

$\alpha_1 = r_2 z_3 - r_3 z_3 = (8 \times 10) - (9 \times 7)$

$\alpha_1 = 17 mm^2$

$\alpha_2 = 3 mm^2$

$\alpha_3 = -14 mm^2$

$\beta_1 = z_2 - z_3 = 7 - 10$

$\beta_1 = -3 mm$

$\beta_2 = 3 mm$

$\beta_2 = 0$

$\gamma_1 = 1 mm$

$\gamma_2 = -3 mm$ $\gamma_3 = 2 mm$

$$r = \frac{r_1 + r_2 + r_3}{3} = \frac{6+8+9}{3}$$

$r = 7.6667$ mm

$$A = \frac{1}{2}\begin{bmatrix} 1 & r_1 & z_1 \\ 1 & r_2 & z_2 \\ 1 & r_3 & z_3 \end{bmatrix}$$

$= \frac{1}{2}\left[1(80-63) - 6(10-7) + 7(9-8)\right]$

$A = 3$ mm

$$B = \frac{1}{2 \times 3}\begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0.2609 & 0 & 0.2609 & 0 & 0.2609 & 0 \\ 0 & 1 & 0 & -3 & 0 & 2 \\ 1 & -3 & 3 & 3 & 2 & 0 \end{bmatrix}$$

$$B^T = 0.1667\begin{bmatrix} 3 & 0.2609 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 3 & 0.2609 & 0 & -3 \\ 0 & 0 & -3 & 3 \\ 0 & 0.2609 & 0 & 2 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

$$D = 0.25 \times 320 \times 10^3 \begin{bmatrix} 3 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$[B^T][D] = 0.1667 \begin{bmatrix} -3 & 0.2609 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 3 & 0.2609 & 0 & -3 \\ 0 & 0 & -3 & 3 \\ 0 & 0.2609 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix} \times 0.25 \times 320 \times 10^3$$

$$\begin{bmatrix} 3 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

= 26.6388	−5.7341	−29.7958	11.21
−5.7391	12	12.2	−18
−29.7933	12.2609	37.7	−18.7
11.2173	−18	−18.78	36
1.4213	5.7341	−5.01	5.2
−5.4782	6	6.52	18

1.41	−5.4
−5.7	6
5.61	6.82
5.21	18
4.30	0.62

$(F) = B^T (D) (e)_t \times 2\pi r A \quad \cdots (1)$

6.

$$(e)_t = 10^{-6} \begin{pmatrix} 150 \\ 150 \\ 0 \\ 150 \end{pmatrix}$$

$$\Rightarrow \begin{bmatrix} F_{1u} \\ F_{1w} \\ F_{2u} \\ F_{2v} \\ F_{3u} \\ F_{3v} \end{bmatrix} = \begin{bmatrix} -2878.25 \\ -289.08 \\ 2903.45 \\ -867.25 \\ 879.86 \\ 1156.34 \end{bmatrix} N$$

$$\Rightarrow \begin{bmatrix} F_{1u} \\ F_{1v} \\ F_{2u} \\ F_{2v} \\ F_{3u} \\ F_{3v} \end{bmatrix} = \begin{bmatrix} -2878.25 \\ -289.08 \\ 2903.45 \\ -867.25 \\ 879.86 \\ 1156.34 \end{bmatrix} N$$

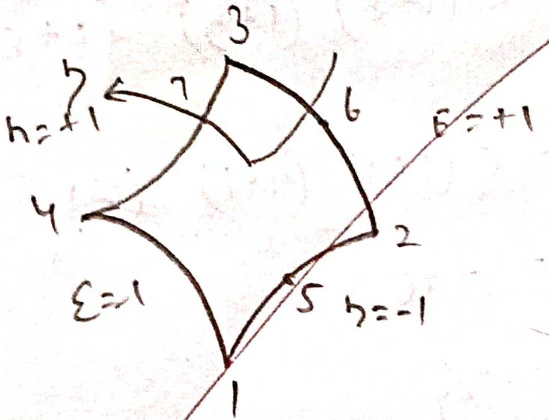
Shape function derivation for the Eight noded rectangular Element

Consider a eight noded rectangular element is shown in fig. 3.24 below serendipity family of elements.

N_1 = 1 at node 1 and 0 at all other nodes

N_1 = 0 at all other nodes

$N_1 = C(1-\varepsilon)(1-\eta)(1+\varepsilon+\eta)$



N_2 = 1 at node 2

N_2 = 0 at all other nodes

$N_2 = C(1+\varepsilon)(1-\eta)(1-\varepsilon+\eta)$

$N_2 = C(1+1)(1+1)(-1)$

$1 = -4C$

$C = -\frac{1}{4}$

$N_1 = -\frac{1}{4}(1+\varepsilon)(1-\eta)(1-\varepsilon+\eta)$

$N_3 = c(1+\varepsilon)(1+\eta)(1-\varepsilon-\eta)$

$N_3 = c(1+1)(1+1)(-1)$

$1 = -4c$

$c = -\frac{1}{4}$

$N_3 = -\frac{1}{4}(1+\varepsilon)(1+\eta)(1-\varepsilon-\eta)$

$N_4 = c(1-\varepsilon)(1+\eta)(1+\varepsilon-\eta)$

$N_4 = c(1+1)(1+1)(-1$

$1 = -4c$

$c = -\frac{1}{4}$

$N_4 = -\frac{1}{4}(1-\varepsilon)(1+\eta)(1+\varepsilon-\eta)$

$N_5 = c(1-\varepsilon)(1-\eta)(1+\varepsilon)$

$N_5 = c(1-\varepsilon^2)(1-\eta)$

$c = \frac{1}{2}$

$N_5 = \frac{1}{2}(1-g)^2(\xi - \eta)$

$N_6 = C(1+\varepsilon)(1-\eta^2)$

$N_6 = C(1+1)(1-0)$

$1 = 2C$

$C = \frac{1}{2}$

$N_6 = \frac{1}{2}(1+\varepsilon)(1-\eta)^2$

$N_7 = C(1+\varepsilon)(1+\eta)(1-\epsilon)$

$N_7 = C(1-\varepsilon^2)(1+\eta)$

$C = \frac{1}{2}$

$N_7 = \frac{1}{2}(1-\epsilon)^2(1+\eta)$

$N_8 = C(1-\varepsilon)(1+\eta^2)$

$N_8 = C(1+1)(1-0)$

$1 = 2C$

$C = \frac{1}{2}$

$N_5 = 1/2\ (1-\varepsilon)\ (1-\eta)^2$

$N_1 = -1/4\ (1-\varepsilon)\ (1-\eta)\ (1+\varepsilon+\eta)$

$N_2 = -1/4\ (1+\varepsilon)\ (1-\eta)\ (1-\varepsilon+\eta)$

$N_3 = -1/4\ (1+\varepsilon)\ (1+\eta)\ (1-\varepsilon-\eta)$

$N_4 = -1/4\ (1-\varepsilon)\ (1+\eta)\ (1+\varepsilon-\eta)$

$N_5 = 1/2\ (1-\varepsilon^2)\ (1-\eta)$

$N_6 = 1/2\ (1+\varepsilon)\ (1-\eta^2)$

$N_7 = 1/2\ (1-\varepsilon^2)\ (1+\eta)$

$N_8 = 1/2\ (1-\varepsilon)\ (1-\eta^2)$