PANDIAN SARASWATHI YADAV ENGINEERING COLLEGE
(Approved by AICTE & Affiliated to Anna University, Chennai)
Madurai - Sivagangai Highway, Arasanoor, Thirumansolai Post, Sivagangai Dt. - 630 561, Tamilnadu
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Internal Assessment Test

Sample paper

[2022-2023]

ODD SEMESTER

PANDIAN SARASWATHI YADAV ENGINEERING COLLEGE

Madurai - Sivagangai Highway, Thirumansolai Post, Arasanoor, Sivagangai - 630 561

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(To be filled in by the candidate)

REGISTER NUMBER	9	1	2	0	2	0	1	1	4	0	0	2

Student Name	R. AKASH		
Degree / Branch	B.E. Mechanical Engineering		
Subject Code	ME8593	Subject Title	Design Of Machine Elements

Year / Semester / Sec	III / V / -	All particulars given are verified
Date & Session	27/09/2022 FN	27/9/22
No. of Pages used		Name and Signature of the Hall Supdt. with date

PART - A		PART - B		Marks				GRAND TOTAL (IN WORDS)
Question No.	Marks	Question No.		I	II	III	Total	
1	2	~~11~~ 8	a	10				
2	2		b					
3	2	~~12~~ 9	a	12				Eighty Six
4	2		b					
5	2	~~13~~ 10	a	7				
6	2		b					
7	2	14	a	29		43/50		GRAND TOTAL
8			b					
9		15	a					86%
10			b					
Total	14							

27/9/22 Date	Signature of the Examiner	Signature of the HOD

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6. Drawings and Sketches should be drawn using pencil.

Factors influencing machine design

* Strength and Stiffness
* Surface finish and tolerances
* Manufactuarability
* Ergonomics and aesthetics
* Working atmosphere
* wear and hardness requirement
* Cooling and lubrication
* Safety and reliability
* Noise requirement
* cost

2) Material properties hardness, Stiffness and resilence

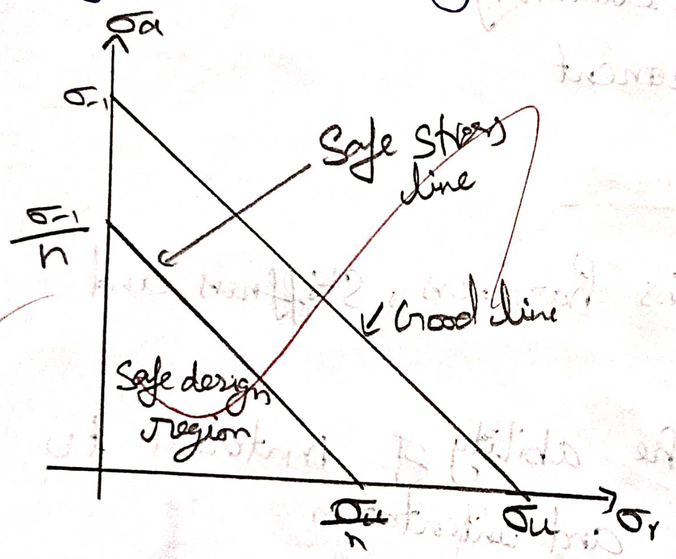
* Hardness is the ability of material to resist Scratching and intentation
* Stiffness is the ability of material to resist deformation under loading.
* Resilence is the ability of material to resist absorb energy and to resist Shock and impact load

3) Stress concentration factor

Stress concentration is the increase in local stresses at points of rapid change in cross section or discontinuities.

* Stress concentration factor K_t is the ratio of maximum stress at critical section to the nominal stress $K_t = \frac{\sigma_{max}}{\sigma_0}$

4) Goodman and Soderberg diagrams and locate the safe design region



5) Function of woodruff key

* A woodruff key is used to transmit less torque in automotive and machine tool industries

* The keyway in the shaft is milled in a curved shape whereas the key way in the hub usually straight.

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6) Equivalent bending moment

when a shaft is subjected to combined bending and torsion loading, the design is usually based on the maximum shear stress theory since the shafts are usually made of ductile materials. The expression $\frac{1}{2}(M_b + \sqrt{M_b^2 + M_t^2})$ is called equivalent bending moment and is denoted by M_{be}

Equivalent bending moment $M_{be} = \frac{1}{2}\left(M_b + \sqrt{M_b^2 + M_t^2}\right)$

diameter of shaft

$$M_{be} = \frac{\pi}{32} \times \sigma_b \times d^3$$

---

7) G.D

$l = 750mm$

$\tau = 40MPa = 40 N/mm^2$

$\theta = 0.017$ radian

$G = 0.8 \times 10^5 MPa = 0.8 \times 10^5 N/m^2$

find: d

Solution:-

$$M_t = \frac{\pi}{16} \times \tau \times d^3$$

$$\theta = \frac{M_t \times l}{GJ} = \frac{\frac{\pi}{16} \times \tau \times d^3 \times l}{G \times \left(\frac{\pi d^4}{32}\right)} = \frac{2\tau l}{Gd}$$

$$0.017 = \frac{2 \times 40 \times 750}{0.8 \times 10^5 \times d}$$

$$\boxed{d = 44.11 \text{ mm}}$$

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8 G.D

$d = 200\ mm$

$W_{min} = -50N$

$W_{max} = 150N$

$n = 2$

$K_t = 1.4$

$q = 0.9$

$\sigma_u = 550\ N/mm^2$

$\sigma_y = 320\ N/mm^2$

$\sigma_{-1} = 275\ N/mm^2$

$K_{SZ} = 0.85$

$K_{ff} = 0.9$

To find:

d - diameter of rod

Solution:

$M_{bmax} = W_{max} \times d = 150 \times 10^3 \times 200 = 3 \times 10^7\ N\text{-}mm$

$M_{bmin} = W_{min} \times d = -50 \times 10^3 \times 200 = -1 \times 10^7\ N\text{-}mm$

$$Z = \frac{\pi d^3}{32}$$

$$\sigma_{max} = \frac{M_{bmax}}{Z} = \frac{3 \times 10^7}{\frac{\pi d^3}{32}} = \frac{0.056 \times 10^8}{d^3}$$

$$\sigma_{min} = \frac{M_{bmin}}{Z} = \frac{-1 \times 10^7}{\frac{\pi d^3}{32}} = \frac{-1.018 \times 10^8}{d^3}$$

$$\sigma_m = \sigma_{max} + \sigma_{min}$$

$$= \frac{3.056 \times 10^8}{d^3} + \left(\frac{-1.018 \times 10^8}{d^3}\right)$$

$$= \frac{2.038 \times 10^8}{d^3}$$

Amplitude Stress $\sigma_a = \sigma_{max} - \sigma_{min}$

$$= \frac{3.056 \times 10^8}{d^3} - \left(-\frac{1.018 \times 10^8}{d^3}\right)$$

$$\sigma_a = \frac{4.075 \times 10^8}{d^3}$$

$$K_f = 1 + q(K_t - 1) = 1 + 0.9(1.4 - 1) = 1.36$$

Using Goodman equation

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_u} + \frac{K_f \cdot \sigma_a}{\sigma_{-1}(K_L \cdot K_{SZ} \cdot K_{SF})}$$

$$\frac{1}{2} = \frac{\frac{2.038 \times 10^8}{d^3}}{550} + \frac{1.36 \times \frac{4.074 \times 10^8}{d^3}}{300(1 \times 0.85 \times 0.9)}$$

d = 177.26 mm

~~Say d = 18 mm~~ 180 MM

Using Soderberg equation

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + \frac{K_f \cdot \sigma_a}{\sigma_{-1}(K_L \cdot K_{SZ} \cdot K_{SF})}$$

$$\frac{1}{2} = \frac{\frac{2.038 \times 10^8}{d^3}}{320} + \frac{1.36 \times \frac{4.074 \times 10^8}{d^3}}{300(1 \times 0.85 \times 0.9)}$$

d = 182.74 mm Say 185 mm

d (Goodman method) = 180 mm

d (Soderberg method) = 185 mm.

10

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9) Given Data:

Tensile load $P = 25 kN = 25 \times 10^3 N$

$F = 10 kN = 10 \times 10^3 N$

$FOS = 2.5$

yield stress $\sigma = 300 N/mm^2$

$\gamma = 0.25$

To find:

Core dia of bolt

Solution:

Stress due to tensile load

$$\sigma_x = \frac{P}{A} = \frac{25 \times 10^3}{(\pi/4 \times d^2)} = \frac{31.83 \times 10^3}{d^2}$$

Stress due to shear load

$$\tau = \frac{F}{A} = \frac{10 \times 10^3}{\frac{\pi}{4} \times d^2} = \frac{12.732 \times 10^3}{d^2}$$

minimum and maximum principle stress

$$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y) + 4\tau^2}$$

$$\tau_{xy} = \tau = \frac{12.732 \times 10^3}{d^2}$$

$$\sigma_x = \frac{31.83 \times 10^3}{d^2}$$

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ADDITIONAL SHEET

Name: Reg.No:

$$\sigma_1 = \frac{1}{2}\left[\frac{31.83\times10^3}{d^2}\right] + \sqrt{\left(\frac{31.83\times10^3}{d^2}\right)^2 + 4\left(\frac{12.732\times10^3}{d^2}\right)^2}$$

$$= \frac{1}{2}\left[\frac{31.83\times10^3}{d^2} + \frac{40.73\times10^3}{d^2}\right]$$

$$\sigma_1 = \frac{36.267\times10^3}{d^2}\ N/mm^2$$

minimum principal stress

$$\sigma_2 = \frac{1}{2}\left[(\sigma_x+\sigma_y) - \sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_y^2}\right]$$

$$= \frac{1}{2}\left[\frac{31.83\times10^3}{d^2} - \frac{40.73\times10^3}{d^2}\right]$$

$$\sigma_2 = \frac{-4.465\times10^3}{d^2}\ N/mm^2$$

maximum principal stress theory

$$\sigma_1 = \frac{\sigma_y}{n} = \frac{36.267\times10^3}{d^2} \quad \sigma_y = 300 \text{ and FOS} = 2.5$$

$$\frac{36.267\times10^3}{d^2} = \frac{300}{2.5}$$

$d = 17.4$ mm say 18 mm

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9)

maximum principal Strain theory

$$\sigma_1 - \gamma(\sigma_2 - \sigma_3) = \frac{\sigma_y}{n} \qquad \sigma_y = 0$$

$$\frac{36.267\times10^3}{d^2} - 0.27\left(\frac{-4.465\times10^3}{d^2}\right) = \frac{300}{2.5}$$

$$\frac{37382S}{a^2} = \frac{300}{2.5}$$

$d = 17.64$ mm . Say 18mm

maximum Shear stress theory

$$\sigma_1 - \sigma_2 = \frac{\sigma_y}{n}$$

$$\frac{36.251\times10^3}{d^2} - \left[\frac{-4.465\times10^3}{d^2}\right] = \frac{300}{2.5}$$

$$\frac{40716}{d^2} = \frac{300}{2.5}$$

$d = 18.42$ mm

Choosing largest of the three diameters.

Core diameter of bolt = 19mm

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ADDITIONAL SHEET

Name: Reg.No:

Given Data:

$T = 250\ N\text{-}m \quad n = 4$

τ_{pa} shaft $= 100 MPa$

(σ_{pr}) shaft $= 250 MPa$

(τ_{pr}) key $= 100 MPa$

(σ_{pr}) key $= 250 MPa$

P_a flange $= 200 MPa$

$(\tau_{per}) = 100 MPa$

To find: Design of rigid flange coupling

Step-1

Calculate the diameter of the Shaft

$$T = \frac{\pi}{16} \times d^3 \times (\tau_{pr})_{shaft}$$

$$250 \times 10^3 = \frac{\pi}{16} \times d^3 \times 100$$

$$d = 23.3508 mm \simeq 24 mm$$

Step.2

Calculate the dimension of the key

(σ_{pr}) key $> 2 (\tau_{pr})$ key

Hence Select a rectangular key

$$w = \frac{d}{4} = \frac{24}{4} = 6 mm$$

$$h = d/6 = \frac{24}{6} = 4 mm$$

$$l = 1.5d = 1.5 \times 24 = 36 mm$$

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length of key is also found by considering shear and crushing stresses

a) considering shear stress

$$T = w \times l \times d/2 \times (\tau_{per})_{key}$$

$$250 \times 10^3 = 6 \times l \times \frac{24}{2} \times 100$$

$$\therefore l = 34.72\ mm$$

b) considering crushing stress

$$T = \frac{h}{2} \times l \times \frac{d}{2} \times (\sigma_{pa})_{key}$$

$$250 \times 10^3 = \frac{4}{2} \times l \times \frac{24}{2} \times 250$$

$$l = 41.667 \simeq 42\ mm$$

Selecting a larger value of $l = 42$ mm

Dimension of key $= 6 \times 4 \times 42$ mm

Step-3

Calculate the dimensions of flange coupling

O.D of hub $D = 2d = 2 \times 24 = 48$ mm

$L = 42$ mm

P.C.D of bolts $D_1 = 3d = 3 \times 24 = 72$ mm

$D_2 = 4d = 4 \times 24 = 96$ mm

$D_3 = 1.1 \times d = 1.1 \times 48 = 53$ mm

Thicke of flange $T = 0.5d = 0.5 \times 24 = 12$ mm

protective flange $t_p = 0.25d$

$= 0.25 \times 24 = 6$ mm

$n = 4$

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ADDITIONAL SHEET

Name: Reg.No:

Step-4

Design check for hub

$$T = \frac{\pi}{16} \times D^3 \times (1-k^4) \times (\tau_{ind})_{hub}$$

$$250 \times 10^3 = \frac{\pi}{16} \times 48^3 \times (1-65)^4 \times (\tau_{ind})_{hub}$$

$$(\tau_{ind})_{hub} = 12.28 \ N/mm^2$$

$$(\tau_{ind})_{max} < (\tau_{per})_{hub}$$

Step-5

Design check for flange.

$$T = \frac{\pi D^2}{2} \times t_f \times (\tau_{ind})_{flange}$$

$$250 \times 10^3 = \frac{\pi \times 48^2}{2} \times 12 \times (\tau_{ind})_{flange}$$

$$(\tau_{ind})_{flange} = 5.7564 \ N/mm^2$$

$5.7564 < 200$ flange is safe.

Step-6

Design of bolts

Bolts are subjected to direct shear and crushing stress

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1	2	11	a	8				
2	2	8	b					
3	2	12	a	8				Seventy Six.
4	2	9	b					
5	2	13	a	8				
6	2	10	b					GRAND TOTAL
7	2	14	a					
8			b	24		38/50		
9	14	15	a					76%
10			b					
Total								

Date 28/10/22	Signature of the Examiner	M. Signature of the HOD

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8) Given data:

$P = 15 \, kN = 15 \times 10^3 \, W$

$N = 200 \, rpm.$

To find:

~~Desinger~~ of the Coupling

Soln

(i) Desigh of Shaft:

$$M_t = \frac{P \times 60}{2\pi N}$$

$$= \frac{15 \times 10^3 \times 60}{2\pi \times 60}$$

$$\boxed{M_t = 716.19 \, N\text{-}m}$$

$$M_t = \frac{\pi}{16} \times \tau_s \times d^3$$

$$716.19 = \frac{\pi}{16} \times 65 \times d^3$$

$$\boxed{d = 38.28 \, mm}$$

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(ii) Dimensions of the coupling

$D = 2.5d = 2.5 \times 40 = 400mm$

$L = 3.5 \times d = 3.5 \times 40 = 140mm$

(iii) Design of sleeve :

$$M_t = \frac{\pi}{16} \times \tau_s \times \left(\frac{D^4 - d^4}{D}\right)$$

~~716.197.2~~

$$716.19 = \frac{\pi}{16} \times \tau_s \times \left(\frac{(100)^4 - (40)^4}{100}\right)$$

$$\boxed{\tau_s = 3.74 \, N/mm^2}$$

(iv) Design of key : Selection of key ?

$$l = \frac{L}{2} = \frac{140}{2}$$

$$\boxed{l = 70mm}$$

$$M_t = l \times b \times \tau_k \times \frac{d}{2}$$

$$716.19 = 70 \times 12 \times \tau_k \times \frac{40}{2}$$

$$\boxed{\tau_k = 42.63 \, N/mm^2}$$

$$M_t = l \times \frac{h}{2} \times \sigma_{ck} \times \frac{d}{2}$$

$$716.19 = 70 \times \frac{8}{2} \times \sigma_{ck} \times \frac{40}{2}$$

$$\boxed{\sigma_{ck} = 127.9\ N/mm^2}$$

(v) Design of bolts :-

$$M_t = \frac{\pi^2}{16} \times \mu \times (d_b)^2 \times \sigma_t \times n \times d$$

$$716.19 = \frac{\pi^2}{16} \times 0.3 \times (d_b)^2 \times 70 \times 4 \times 40$$

$$d_b^2 = 345.52$$

$$\boxed{\begin{array}{l} d_b = 18.594\ mm \\ d_b = 20\ mm \end{array}}$$

8

---

10) Given data :-

$$P = 70\ kN = 70 \times 10^3\ N$$

$$S_{sy} = 396\ N/m^3$$

$$fos = 6$$

$$S_{yt} = 420\ N/mm^2$$

To find

Design a knuckle joint

Step 1:

$$\sigma_t = \sigma_c = \frac{S_{yt}}{fos} = \frac{420}{6}$$

$$\boxed{\sigma_t = \sigma_c = 70\ N/mm^2}$$

$$\tau = \frac{S_{sy}}{fos} = \frac{396}{6} = 66\ N/mm^2$$

$$\sigma_t = \frac{P}{\frac{\pi}{4} \times d^2}$$

$$70 = \frac{70 \times 10^3}{\frac{\pi}{4} \times d^2}$$

$$\boxed{d = 35.6824\ mm = 40mm}$$

Step 2

$$\tau = \frac{P}{2 \times \pi/4 \times d_p^2}$$

$$66 = \frac{70 \times 10^3}{2 \times \pi/4 \times (d_p)^2}$$

$$\boxed{d_p = 25.9846 = 30mm}$$

$d_n = 1.5\,d = 1.5 \times 40$

$\boxed{d_n = 60\,mm}$

Step 3

$$\sigma_c = \frac{P}{2\,d\,p\,t_1} = \frac{70 \times 10^3}{2 \times 30 \times t_1}$$

$\boxed{t_1 = 16.666 = 17\,mm}$

$t_1 = 0.75d = 0.75 \times 40$

$\boxed{t_1 = 30\,mm}$

$t_2 = 0.5 \times d = 0.5 \times 40$

$\boxed{t_2 = 20\,mm}$

Step 4

$$\sigma_c = \frac{P}{d\,t}$$

$$70 = \frac{70 \times 10^3}{30 \times t}$$

$\boxed{t = 33.333\,mm}$

$\boxed{t = 34\,mm}$

$t = 1.25 \times d = 1.25 \times 40 = 50\,mm$

$\boxed{t = 50\,mm}$

Step 5

$$\sigma_t = \frac{P}{(D-d_p)t}$$

$$70 = \frac{70\times10^3}{(D-30)\times50}$$

$$D = \frac{70\times10^3}{70\times50} + 30$$

$$\boxed{D = 50mm}$$

$$\tau = \frac{P}{(D-d_p)t}$$

$$66 = \frac{70\times10^3}{(D-30)\times50}$$

$$= \frac{70\times10^3}{66\times70} + 30$$

$$= 51.2121mm$$

$$\boxed{D = 55mm}$$

Step 6

$$\sigma_t = \frac{P}{2(D-d_p)t_1} = \frac{70\times10^3}{2(55-30)\times30}$$

$$\sigma_t = 46.66\ N/mm^2 < 70N/mm^2$$

$$\tau = \frac{P}{2(D-d_p)t}$$

$$\tau = 46.66 < 66N/mm^2$$

① ADDITIONAL SHEET

Name: Reg.No:

9) Given data :

$P = 2000\,N$

$e = 200\,mm$

$l_1 = 50\,mm$

$l_2 = 250\,mm$

$n_1 = 2$

$n_2 = 3$

To find

Ball size 'd'

Soln

$$f_1 = \frac{P \cdot e \cdot l_1}{n_1 \cdot l_1^2 + n_2 \cdot l_2^2}$$

$$= \frac{20000 \times 200 \times 50}{(2 \times 50^2) + (3 \times 250^2)}$$

$$\boxed{f_1 = 1038.96\,N}$$

$$f_2 = \frac{P \cdot e \cdot l_2}{n_1 \cdot l_1^2 + n_2 \cdot l_2^2}$$

$$= \frac{20000 \times 200 \times 250}{(2 \times 50^2) + (3 \times 250^2)}$$

$$\boxed{F_2 = 5194.8 N}$$

$F_2 > F_1$

$$\sigma_2 = \frac{F_2}{A_c} = \frac{5194.8}{A_c}$$

$$\tau = \frac{P}{(n_1+n_2)A_c} = \frac{20000}{(2+3)\,A_c}$$

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2} = \frac{1}{2}\sqrt{\left(\frac{5194.8}{A_c}\right)^2 + 4\left(\frac{4000}{A_c}\right)^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{5194.8}{A_c}\right)^2 + 4\left(\frac{4000}{A_c}\right)^2}$$

$$\tau_{max} = \frac{4768.65}{A_c}$$

$$\tau_{max} = \frac{\sigma_y}{n}$$

Assuming $\sigma_y = 300\ N/mm^2$ and safety factor $= 3$

$$\frac{4768.65}{A_c} = \frac{300}{3}$$

$$A_c = 47.68\ mm^2$$

$$A_c = 47.68\ mm^2 (\simeq 58 mm^2)$$

Bold chosen M10 X 1.5 bolt.

8

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②

ADDITIONAL SHEET

Name: Reg.No:

Part - A

1) 1) Rignd cuplings :-

* Sleeve Cauplings
* flangs caplings
* clemp caupling

2) Flexible Cauplings :-

* universel Cauplings
* aldhams's Cauplings
* Pushed pin type Cauplings

2

S NO	Keys	Splines
1	A shaft which is haing single keyway	A shaft which is haing multiple keyways
	Keys are used in Cauplings.	Splines are used in automobiles and mechine tools.

2

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3)

* Saddle key
* Tangent key
* Sunk key
* Round key and taper pins.

4)

* High clamping
* Small tightening force requirement.
* Easy manufacturing
* Simple design.

5)

* welded connections subjected to moment in a plane of the weld.
* welded connections subjected to moment in a plane normal to the plane of the weld.

6)

Soln

from PSGDB 5.42, for M20

$A_c = 245\ mm^2$

$\sigma = \frac{P}{A_c} = P = \sigma \times A_c$

ADDITIONAL SHEET

⑬

Name: Reg.No:

$\cancel{Ae} = 40 \times 245$

$\boxed{P = 9800\ N}$

7) * A thread is designed with

(i) Letter 'M' followed by

(ii) nominal diameter in mm and

(iii) pitch in mm [for fine pitches only]

$\boxed{Md \times p}$

* If coarse pitches are used then 'p' value is omitted. Thus M20 x 2.5 means

(i) Nominal diameter is 20mm

(ii) 2.5 mm pitch, fine thread.

* M20 means, 20mm nominal diameter with coarse threads.

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PART - A		PART - B						GRAND TOTAL (IN WORDS)
Question No.	Marks	Question No.		Marks I	II	III	Total	
1	2	11	a	11				Eighty one.
2	2		b					
3	1	12		11.				
4	2		b					
5	2	13	a	10.				
6	2		b					
7	2	14	a	10.		81/100		GRAND TOTAL
8	2		b					
9	1	15	a	9.				81%.
10	2		b					
Total	18	16	A	12	63			

25/11/22 Date	Signature of the Examiner	M. Nadarajan Signature of the HOD

Instructions to the Candidates :

1. Write your Register No. in the type as shown in the following example

6	2	0	1	1	3	7	5	1	0	2	1

2. Write your Register Number at the Top Right Hand Side of the QUESTION PAPER
3. Use both sides of the paper for answering questions.
4. Possession of any incriminating material and malpractice of any nature shall be punishable as per rules.
5. Answers must be legibly written in ink (Blue, Black or Blue Black)
6. Drawings and Sketches should be drawn using pencil.

Part-C

16

b)

Given Data:

Scale on x-axis for crank angle.

$\theta = 1\,mm = 3.6^\circ = \frac{3.6 \times \pi}{180}$

Scale on y-axis for Torque.

$T = 1\,mm = 200\,Nm$

$N = 480\,rpm$

$b = 2.5t$

$c = 0.9$

$\rho = 7250\,kg/m^3$

$\sigma_{fr} = 5\,mpa.$

Driving torque = unable.

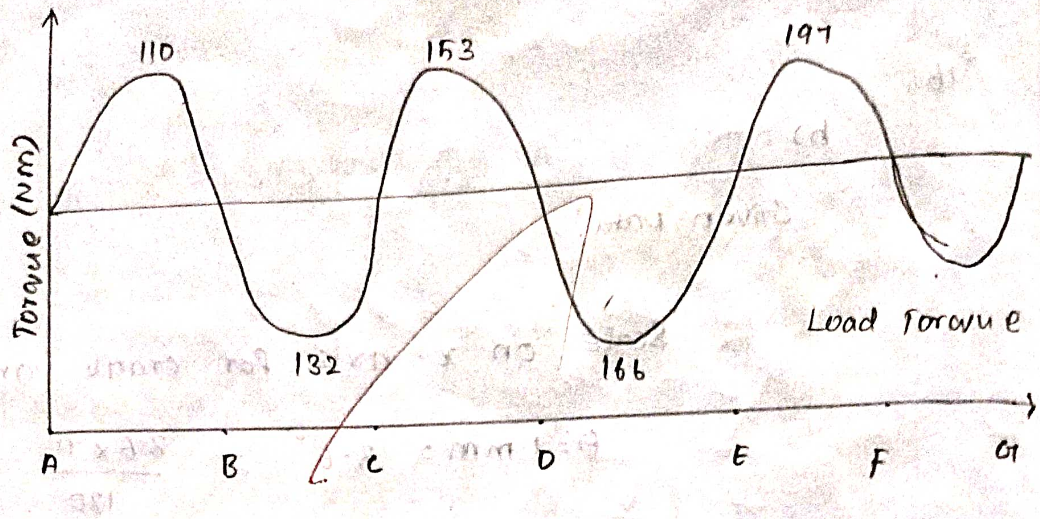
Load torque = constant.

To Find:

$R_m = ?$, $b = ?$, $t = ?$

Solution.

Step: 1 Plot the $T-\theta$ Diagram of Sine cas of Load.



Step : 2 Calculate max flucutuation of Energy.

The scale for on T - θ diagram.

$$1mm^2 = \frac{3.6 \times \pi}{180} \times 200$$

$$\boxed{= 12.56635}$$

unit ?

Let Energy stroted in the flywheel at point A = E

$B = E + 110$

$C = E + 110 - 132 = E - 22.$

$D = E - 22 + 153 = E + 131$

$E = E + 131 - 166 = E - 35$

$F = E - 35 + 197 = E + 162$

$G = E + 162 - 162 = E$

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There fore, max energy in Flywheel is at point, $\boxed{E_{max} = E + 162}$

Min Energy in Flywheel is at point

$\boxed{E_{min} = E - 35}$

max fluctuation of Energy,

$$\Delta E_{max} = E_{max} - E_{min}$$

$$\Delta E_{max} = (E + 162) - (E - 35)$$

$$= 197 \times 12.5663$$

$$\boxed{\Delta E_{max} = 2475.5611 \text{ J}}$$

Step: 3

Select co-efficient of fluctuation of speed (C_s)

W.K.T,

$$C_s = \frac{\omega_{max} - \omega_{min}}{\omega}$$

$$= \frac{10}{480} = 0.0208 \quad [\because \omega_{max} - \omega_{min} = 10]$$

Mean angular speed, $\omega = \frac{2\pi N}{60}$

$$\boxed{\omega = 50.2654 \text{ rad/sec.}}$$

$\Delta E_{max} = I\omega^2 \cdot C_s$

$2475.5611 = I \times (50.2654)^2 \times 0.0208$

$\boxed{I = 47.1053 \ kg\text{-}m^2}$

Now, mass moment of Inertia of rim

$I_r = C \times I$

$= 0.9 \times 47.1053$

$\boxed{I_r = 42.3948 \ kg \cdot m^2}$

Step : 4

Select material for flywheel and define mass Density (ρ) and Limiting rpm Speed.

$\rho = 7250 \ kg/m^3$

w.k.T,

$\sigma_{tr} = \rho v^2$

$5 \times 10^6 = 7250 \times v^2$

$\boxed{v = 26.2612 \ m/s}$

$V = L.R.S \leq 26.2612 \ m/s.$

Step : 5

Calculate mean Radius of Rm.

$V = L.R.S = Rm \cdot \omega$

$V \geq Rm \cdot \omega$

$26.2612 \geq Rm \times 50.2654$

$Rm \leq 0.5224$ m.

$\boxed{Rm = 520 \text{ mm.}}$

Step: 6

Calculate width (b) and thickness (t) of rim.

W.K.T,

$I_r = m \cdot Rm^2 = (2\pi Rm \cdot b \cdot t \cdot \rho) \times Rm^2$

$42.3948 = 2\pi (0.52)^3 \times (t \times 2.5t) \times 7250$

$t^2 = 1.3767 \times 10^{-3}$

$\boxed{t = 0.0371 \text{ m.}}$

$t = 37.1042$ (or) 38 mm.

$b = 2.5t = 2.5 \times 38 = 95$ mm.

Step: 7

Size of the rimmed fly wheel.

$Rm = 520$ mm. [mean Radius of rim]

$Dm = 1040$ mm. [mean Diameter of rim]

$b = 38$ mm.

$t = 95$ mm.

12

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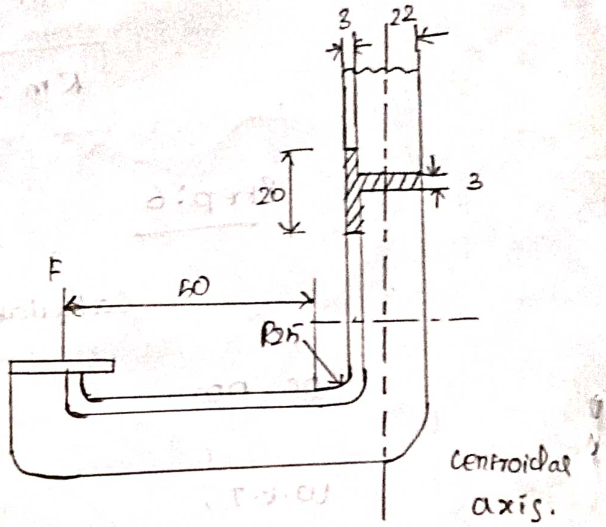
Part - B.

11

a) Given Data!

σ_{max} = 180 MPa.



To Find:

Maximum Force, F.

Solution:

For T section.

$$r_n = \frac{(b_i - t)t_i + t \times h}{(b_i - t)\ln\left[\frac{r_i + t_i}{r_i}\right] + t\ln\left(\frac{r_o}{r_i}\right)}$$

In this case,

$b_i = 20$, $t = 3$, $t_i = 3$

$h = 22$, $r_i = 25$. $r_o = 25 + 3 + 22 = 50$ mm

$$r_n = \frac{(20-3)3 + (3 \times 22)}{(20-3)\ln\left(\frac{25+3}{25}\right) + 3\ln\left(\frac{50}{25}\right)}$$

$$\boxed{r_n = 29.2 \text{ mm.}}$$

Also, For a T section.

$$R = r_i + \frac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b_i - t)}{ht + (b_i - t)t_i}$$

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ADDITIONAL SHEET

Name: Reg.No:

Substituting the values, we have.

$$R = 25 + \frac{\left[\frac{1}{2} \times 22^2 \times 8\right] + \frac{1}{2} \times 8^2 \times (20-8)}{(22 \times 8) + (20-3)3}$$

$$\boxed{R = 31.85 \text{ mm.}}$$

$$e = R - r_n = 31.85 - 29.2$$

$$\boxed{e = 2.65 \text{mm.}}$$

$$h_i = r_n - r_i = 29.2 - 25$$

$$\boxed{h_i = 4.2 \text{ mm.}}$$

Area, $A = (20 \times 8) + (22 \times 8)$

$$\boxed{A = 126 \text{mm}^2}$$

Bending moment, $M_b = F \times (50 + 31.85)$

$$\boxed{M_b = 81.85 \, F \text{ N.mm}}$$

w.k.T, bending stress.

$$\sigma_b = \frac{M_b \times h_i}{a \cdot e \cdot r_i} = \frac{81.85 \times 4.2}{126 \times 2.65 \times 25}$$

$$\boxed{\sigma_b = 0.0412 \, F \text{ N/mm}^2.}$$

Direct stress.

$$\sigma_d = \frac{F}{A}$$

$$\boxed{\sigma_d = \frac{F}{126}\ N/mm^2.}$$

Maximum stress,

$$\sigma_{max} = \sigma_b + \sigma_d = 0.0412\,F + \frac{F}{126}$$

$$\boxed{\sigma_{max} = 0.04914\ F\ N/mm^2.}$$

Equating this to the maximum permissible tensile stress we have.

$$0.04914\ F = 130$$

$$\boxed{F = 2645.69\ N}$$

$$\boxed{F = 2.645\ kN.}$$

Result:

Maximum force, F = 2.645 kN.

---

12 b)

Given Data:

$T = 250$ N·m

$n = 4$

$(\tau_{pa})_{shaft} = 100$ mpa.

$(\sigma_{per})_{shaft} = 250$ mpa

$(\tau_{per})_{key} = 100$ mpa.

$(\sigma_{per})_{key} = 250$ mpa.

$(\tau_{per})_{flange} = 200$ mpa.

$(\tau_{per})_{bolt} = 100$ mpa.

ADDITIONAL SHEET

Name: Reg.No:

To find:

Design of rigid flange coupling.

Solution:

Step: 1

Calculate the diameter of the shaft.

W.K.T

$$T = \frac{\pi}{16} \times d^3 \times (\tau_{per})_{shaft}$$

$$250 \times 10^3 = \frac{\pi}{16} \times d^3 \times 100$$

$$d = 23.3508 \text{ mm}$$

$$\boxed{d = 24 \text{ mm.}}$$

Step: 2

Calculate the Dimensions of the key.

It is given that,

$$(\sigma_{per})_{key} > 2(\tau_{per})_{key}.$$

Hence select a Rectangular key.

For Rectangular key,

$$w = \frac{d}{4} = \frac{24}{4} = 6 \text{mm.}$$

$$h = \frac{d}{6} = \frac{24}{6} = 44 \text{ mm.}$$

$$l = 1.5d = 1.5 \times 24 = 36 \text{ mm.}$$

Length of key is also found by considering shear and crushing stressed.

a) considering shear stress.

$$T = w \times l \times \frac{d}{2} \times (\tau_{per})_{key}.$$

$$250 \times 10^3 = 6 \times l \times \frac{2.4}{2} \times 100$$

$$\boxed{l = 34.722 \text{ mm.}}$$

b) considering crushing stress.

$$T = \frac{h}{2} \times l \times \frac{d}{2} \times (\tau_{per})_{key}.$$

$$250 \times 10^3 = \frac{4}{2} \times l \times \frac{24}{2} \times 250$$

$$l = 41.667 \text{ mm}$$

$$\boxed{l = 42 \text{ mm}}$$

Selecting a Larger value of $l \times d = 42$ mm.

Dimension of key $= 6 \times 4 \times 42$ mm.

Step :3

Calculate the Dimension of flange coupling.

O.D of hub, $D = 2d = 2 \times 24 = 48$ mm.

Length of hub, $L =$ Length of key $= 42$ mm.

P.C.D of bolts, $D_1 = 3d = 3 \times 24 = 72$ mm.

O.D of flange, $D_2 = 4d = 4 \times 24 = 96$ mm.

Dia of flange, $D_3 = 1.1D = 1.1 \times 48 = 52.8$ mm.

ADDITIONAL SHEET

Name: Reg.No:

Thickness of flange, $t_f = 0.5d = 0.5 \times 24 = 12\,mm$.

Thickness of protective flang, $t_p = 0.25d$

$= 0.25 \times 24 = 6\,mm$.

Number of bolts, $n = 4$

Step 4:

Design chuck for Hub.

Torque transmitted by hub is.

$$T = \frac{\pi}{16} \times D^3 \times (1 - k^4) \times (\tau_{por})hub.$$

$$250 \times 10^3 = \frac{\pi}{16} \times 48^3 (1 - 0.5^4) \times (\tau_{por})hub.$$

$$\boxed{(\tau_{por})hub = 12.2804\,N/mm^2.}$$

$(\tau_{por})hub < (\tau_{por})hub.$

Step: 5

Design chuck for flange.

$$T = \frac{\pi D^2}{2} \times t_f \times (\tau_{ind})flange.$$

$$250 \times 10^3 = \frac{\pi \times 48^2}{2} \times 12 \times (\tau_{ind})flange.$$

$$\boxed{(\tau_{ind})flange = 5.7564\,N/mm^2}$$

$5.76 < 200$, Flange are safe.

Step 6:

Design of bolts.

Bolt are subjected to direct shear and crushing stresses.

a) considering shearing of bolts.

$$T = (\tau_{per})_{bolt} \times \frac{\pi}{4} \times d_b^2 \times n \times \frac{b}{2}$$

$$250 \times 10^3 = 100 \times \frac{\pi}{4} \times d_b^2 \times 4 \times \frac{48}{2}$$

$$d_b = 5.7582 \text{ mm.}$$

$\boxed{d_b = 6mm,}$ bolts are safe.

b) considering crushing of bolts.

$$T = n \times d_b \times t_f \times \frac{D_1}{2} \times (\sigma_{ind})_{bolt}.$$

$$250 \times 10^3 = 4 \times 6 \times 12 \times \frac{72}{2} \times (\sigma_{ind})_{bolt}.$$

$$\boxed{(\sigma_{ind})_{bolt} = 24.1126 \text{ N/mm}^2}$$

$(\sigma_{ind})_{bolt} < (\sigma_{per})_{bolt}$ is safe.

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ADDITIONAL SHEET

Name: **Reg.No:**

13 a)

Given Data :

$p = 10\,kN = 10000\,N.$

$n = 4$

$e = 500\,mm.$

$x = 100\,mm.$

$y = 75\,mm.$

$\tau = 80\,N/mm^2.$

To find:

Design the size of bolt.

Solution:

$F_1 = \frac{P}{n} = \frac{10 \times 10^3}{4}$

$\boxed{F_1 = 2500\,N.}$

$r = \sqrt{x^2 + y^2} = \sqrt{100^2 + 75^2}$

$\boxed{r = 125}$

$F_2 = \frac{Pe}{nr} = \frac{10 \times 10^3 \times 500}{4 \times 125}$

$\boxed{F_2 = 10000\,N}$

$$\cos\theta = \frac{x}{r} = \frac{100}{125}$$

$$\boxed{\theta = 36.86^\circ}$$

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{2500^2 + 10000^2 + 2\times 2500\times 10000 \times \cos 36.86}$$

$$\boxed{F_R = 12093\,N.}$$

Equating the applied and allowable stresses,

$$\frac{12093}{A_c} = 80$$

$$\boxed{A_c = 151.8\,mm^2}$$

From PSGDB for stress area = 151.8 mm2

bolt chosen is M16.

14

a)

Given Data:

$N_1 = 2.5\,kN = 2.5\times10^3\,N.$

$N_2 = 3.5\,kN = 3.5\times10^3\,N.$

$\delta = 5\,mm.$

$C = 5$

$\sigma_u = 1050\,mPa$

$G = 81370\,mPa.$

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ADDITIONAL SHEET

Name: Reg.No:

$\tau = 0.5\,\sigma_{ut}$

To Find:

$d = ?$, $D = ?$, $n = ?$, $n' = ?$

$L_s = ?$, $L_f = ?$, u required $= ?$

u actual $= ?$

Solution:

Step 1:

Calculate the wire dia and mean coil diameter.

$$\tau = 0.5\,\sigma_{ut} = 0.5 \times 1050$$

$$\boxed{\tau = 525\ N/mm^2}$$

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$= \frac{4(5)-1}{4(5)-4} + \frac{0.615}{5}$$

$$\boxed{K_w = 1.3105.}$$

$$\tau = K_w \left[\frac{8WD}{\pi d^3}\right]$$

$$= K_w \left[\frac{8WC}{\pi d^2}\right]$$

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$$525 = (1.3105)\left[\frac{8 \times 3500 \times 5}{\pi \times d^2}\right]$$

$d = 10.5469$ mm.

$$\boxed{d = 11 \text{ mm.}}$$

mean coil diameter, is $D = c \times d$

$= 5 \times 11$

$$\boxed{D = 55 \text{ mm.}}$$

$$\boxed{\delta = 8.64 \times 10^{-3} W \cdot \text{mm.}}$$

As the workdone by the falling Load is Equal to the energy stored in helical springs.

$$P(h+\delta) = \frac{1}{2} \times W \times \delta \times \text{no. of springs.}$$

$$60 \times 10^3 (50 \times 10^3 + 8.64 \times 10^{-3} W) = \frac{1}{2} \times W \times 8.64 \times 10^{-3} W \times 10$$

$$3 \times 10^9 + 518.4 W = 0.0432 W^2$$

$$0.0432 W^2 - 518.4 W - 3 \times 10^9 = 0$$

Solving the above eqn,

$W = -257.507 \times 10^3$ N

$$\boxed{W = 269.507 \times 10^3 \text{ N.}}$$

ADDITIONAL SHEET

Name: Reg.No:

Selecting positive values.

$w = 269.507 \times 10^3 N.$

Step 2:

calculate the max. shear stress reached in each spring.

$$k_w = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$

$$= \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6}$$

$$\boxed{k_w = 1.2525.}$$

Maximum shear stress.

$$\tau_{max} = \frac{8wc}{\pi d^2} \times k_w$$

$$= \frac{8 \times 269.507 \times 6}{\pi \times 50^2} \times 1.2525$$

$$\boxed{\tau_{max} = 2063 \text{ mpa}.}$$

10

15

a)

Given Data:

$N = 3.2\ kN = 3.2 \times 10^3 N.$

$N_s = \frac{1490}{60}$ RPS

$d = 50mm$

$l = 50mm$

$c = 0.05mm$

$\mu = 25 \times 10^{-9} \frac{N.s}{mm^2}$

To Find:

1) S 2) P_f 3) h_0 4) θ , 5 ΔT

Solution:

Step:1

Load per unit projected ques (P)

$p = \frac{W}{ld} = \frac{3200}{50 \times 50}$

$\boxed{P = 1.28\ N/mm^2}$

Step:2

Summerfeld number (S)

$S = \left(\frac{r}{c}\right)^2 \times \left(\frac{\mu . n_s}{p}\right)$

$= \left(\frac{25}{0.05}\right)^2 \times \left(\frac{25 \times 10^{-9} \times (1490/60)}{1.28}\right)$

$\boxed{S = 0.1212.}$

ADDITIONAL SHEET

Name: Reg.No:

Step3:

Dimensionless parameters.

we get following information S Ud

t	(h_0/c)	ϕ (ϕ_s)	S	$\frac{Q}{rc \cdot n_s \cdot d}$	$\frac{Q_s}{Q}$	$\frac{P}{P_m}$
			0.12212	1		
0.6	0.4	50.38	3.22	4.33	0.600	24.05

Step4:

Coefficient of friction. (F)

$$\left(\frac{r}{c}\right) F = 3.22$$

$$\left(\frac{25}{0.05}\right) F = 3.22$$

$$\boxed{F = 0.0644}$$

Step5:

Power Lost in Friction (P_f)

$$P_f = \frac{(2\pi \cdot n_s)\ (f \cdot w \cdot r)}{10^6}$$

$$= \frac{2\pi \left(\frac{1460}{60}\right) \times (0.644 \times 8200 \times 25)}{10^6}$$

$$\boxed{P_f = 0.08\ kw.}$$

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Step 6:

Minimum oil thickness. (h_o)

$$\left(\frac{h_o}{c}\right) = 0.4$$

$$h_o = 0.4 \times 0.05$$

$$\boxed{h_o = 0.02\,mm.}$$

Step 7:

Total flow rate of Lubricant (Q)

$$\left(\frac{Q}{r.c.n_s.l}\right) = 4.33$$

$$\left[\frac{Q}{25 \times 0.05 \times \frac{1490}{60} \times 50}\right] = 4.33$$

$$Q = 6720\,mm^3/s$$

$$= 6720 \times 52 \times 10^{-9} \times 60 \quad mm^3/mm$$

$$\boxed{Q = 0.4032\ lit/m}$$

Step 8:

Temperature rise of Lubricant (ΔT)

Neglecting offset of side Leakage.

$$\Delta T = \frac{(2\pi n_s)(F.\omega.r)}{\delta.Q.C_p} \times 10^3$$

Assuming. $\delta = 860\ kJ/m^3$

$C_p = 1.76\ kJ/kg.C.$

9

ADDITIONAL SHEET

Name: Reg.No:

$$= \left(2\pi \times \frac{1490}{60}\right) \times \frac{(0.0664 \times 8200 \times 25)}{860 \times 6720 \times 5 \times 2 \times 1.76} \times 10^3$$

$\boxed{\Delta T = 7.9027^\circ c}$

Part - A.

1 * Hardness :

*It is the ability of material to resist scratching and indentation.

* Stiffness :

* It is the ability of material to resist deformation under loading.

* Resilience :

* It is the ability of material to resist absorb energy and to resist shock and impact load.

2 Unilateral :

* A unilateral tolerance is tolerance in which variation is permitted only in one direction from the specified

direction . ex : 1800 + 0.000/ -0.060.

Bilateral :

Bilateral tolerance is tolerance in which variation is permitted in both direction from the specified direction . ex : $1800^{+0.06/-0.06}$

3

S.NO	Keys	Splines.
1	A Shaft which is having single keyway	A shaft which is having multiple keyway.
2	Keys are used in coupling	Splines are used in automobiles and machine tools.

4 Critical speed :

The speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite is known as critical or whirling speed.

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5 Disadvantages of welding:

*It has poor vibration damping characterstics.

*welding result is a thermal distoration of the parts, thereby including residual stresses. Therefore it needs stress relieving heat treatment.

* The Quality and strength of welded joints also depend upon the skill of the labours.

6 Tap bolts.

*One of the parts being joined has enough thickness to accommadate a threaded hole.

* Insufficient space for a nut.

* Material is strong enough so that the threads have long life.

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7 Fluctuation of

* The difference between maximum speed and minimum speed during a cycle is called maximum fluctuation of speed.

* The ratio of maximum fluctation of speed to the mean speed is called co-efficient of fluctuation of speed k.

$$\boxed{\Delta E = E_{max} - E_{min}.}$$

* the ratio of fluctuation of energy to the mean energy is called coefficient of fluctuation of energy;

$$k_E = \frac{E_{max} - E_{min}}{E} = \frac{\Delta E}{E}$$

8

S.no	Open coiled springs	Closed coiled springs.
1	The wires are coiled such that there is a gap between the two consecutive turns.	The spring wires are coiled very closely, each turn is nearly at right angles to the axis
2	Helix angle is large (>10°)	Helix angle is less than 10°

ADDITIONAL SHEET

Name:

Reg.No:

3	In open coiled springs, angles of helix is Large.	In closed coiled springs, angle of helix is small.
4	both torsional and bending stress are significant.	Only torsional stresses are predominant.

9 Advantages of hydrodynamic bearing.

* The contact surfaces must meet at a slight angle to allow the formation of the Lubricant wedge.

* The fluid must be adhering to contact surfaces for conveyance into the pressure area to support the Load.

* The fluid must be distributing itself completely within the bearing clearance area.

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10

* Starting Friction is Low.

* Lubrication is simple.

* It requires Less axial space and more diameteral space.

* Heavier Loads and higher speeds are permissible.

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PANDIAN SARASWATHI YADAV ENGINEERING COLLEGE

(Approved by AICTE & Affiliated to Anna University, Chennai)

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Internal Assessment Test

Sample paper

[2022-2023]

EVEN SEMESTER

PANDIAN SARASWATHI YADAV ENGINEERING COLLEGE

Madurai - Sivagangai Highway, Thirumansolai Post, Arasanoor, Sivagangai - 630 561

Read the instructions given below carefully before filling in the title page
(To be filled in by the candidate)

REGISTER NUMBER	9	1	2	0	2	0	1	1	4	0	0	1

Student Name	AJAY GOWTHAM.		
Degree / Branch	B.E - Mechanical Engineering		
Subject Code	ME 8692	Subject Title	FINITE ELEMENTS AND ANALYSIS

Year / Semester / Sec	III / VI.
Date & Session	04-03-2023-FN
No. of Pages used	15

All particulars given are verified

Name and Signature of the Hall Supdt. with date

PART - A		PART - B						GRAND TOTAL (IN WORDS)
Question No.	Marks	Question No.		Marks I	II	III	Total	
1	2	~~11~~ 8	a	10				
2	2		b					Eighty four.
3	2	~~12~~ 9		9				
4	2		b					
5	2	~~13~~ 10	a	9				
6	2		b					GRAND TOTAL
7	2	14	a	28		62/70		
8	14		b					
9		15	a					84%.
10			b					
Total								

5/3/23 Date	Signature of the Examiner	M. [signature] Signature of the HOD

Instructions to the Candidates :

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3. Use both sides of the paper for answering questions.
4. Possession of any incriminating material and malpractice of any nature shall be punishable as per rules.
5. Answers must be legibly written in ink (Blue, Black or Blue Black)
6. Drawings and Sketches should be drawn using pencil.

PART-A

1.) Finite element method is a numerical method for solving problems of engineering and mathematical physics.

In the finite element method, instead of solving the problem for the entire body in one operation.

2.) i) force method.

ii) Stiffness method.

3.) *) It is easy to formulate and computerize.

*) It is easy perform differential or integration.

*) The accuracy of the results can be improved.

4.) Analysis and evaluation of the solution results is referred to as post processing. Post Processor computer programs help the used to interpret the results by displaying them in graphical form.

5.) Rayleigh - Ritz method is a integral approach method which is useful for solving complex structural problem, encountered in finite element analysis.

6.) In finite element method, field variables within an element are generally expressed by the following approximate relation.

$$\phi(x,y) = N_1(x,y)\,\phi_1 + N_2(x,y)\,\phi_2 + N_3(x,y)\,\phi_3$$

ϕ_1, ϕ_2, ϕ_3 are the values is the field variable and N_1, N_2, N_3 are the interpolation function.

7.) When the inertia effect due to the mass of the components is also considered in addition to the externally applied load, then the analysis is called dynamic analysis.

PART - B

8.) Given Data:

Differential equation, $\frac{d^2y}{dx^2} + 50 = 0$ → ①

$x \le x \le 10$

Trial function, $y = a_1 x (10 - x)$

Boundary conditions are, $y(0) = 0$

$y(10) = 0$.

Solution:

Trail function is $y = a_1 x (10 - x)$

$x = 0, \; y = 0$

$x = 10, \; y = 0$

i) Point collacation method:

$y = a_1 x (10 - x)$

$y = a_1 (10x - x^2)$

$dy/dx = a_1 (10 - 2x)$

$\frac{d^2y}{dx^2} = -2a_1$

sub $\frac{d^2y}{dx^2}$ value in differential equ ①.

$\Rightarrow R = -2a_1 + 50$ → ②

$\Rightarrow R = -2a_1 + 50 = 0$

$-2a_1 = -50$

$a_1 = 25 \rightarrow (3)$

$y = 25x(10-x)$

(ii) Subdomain collaocation method:

method requires, $\int_0^{10} R\,dx = 0$

Sub in R value, $\Rightarrow \int_0^{10} [-2a_1 + 50]\,dx = 0$

$= \int_0^{10} [-2a_1\,dx + 50\,dx] = 0$

$= [-2a_1 x + 50x]_0^{10} = 0$

$= -2a_1(10) + 50(10) - [0] = 0$

$\Rightarrow -20a_1 = -500$

$a_1 = 25 \rightarrow (4)$

$y = 25x(10-x).$

(iii) Least squares method:

$I = \int_0^{10} R^2\,dx.$

$\frac{dI}{da_1} = \int_0^{10} R\,\frac{dR}{da_1}\,dx \rightarrow (5)$

$R = -2a_1 + 50$

$\partial R / \partial a_1 = -2$

sub R and $\partial R / \partial a_1$ value equ (5),

$$\Rightarrow \frac{\partial I}{\partial a_1} = \int_0^{10} (-2a_1 + 50)(-2)\,dx.$$

$$= \int_0^{10} (-2a_1 + 50)(-2)\,dx = 0$$

$$= \int_0^{10} (-2a_1 + 50)\,dx = 0$$

$$= \int_0^{10} [-2a_1\,dx + 50\,dx] = 0$$

$$= [-2a_1 x + 50x]_0^{10} = 0$$

$$= -2a_1(10) + 50(10) - [0] = 0$$

$$\Rightarrow -20a_1 + 500 = 0$$

$$a_1 = 25 \rightarrow (6)$$

$$y = 25x(10-x)$$

(iv.) Galerkin's method:

$$\Rightarrow \int_0^{10} W_1 R\,dx = 0 \rightarrow (7)$$

$$y = W_1 = a_1 x(10-x)$$

Sub w and R values in equ (4),

$$= \int_0^{10} a_1 x(10-x) \times (-2a_1+50)\,dx = 0$$

$$= a_1 \int_0^{10} (10x-x^2)(-2a_1+50)\,dx = 0$$

$$= a_1 \int_0^{10} [-20a_1 x + 500x + 2a_1 x^2 - 50x^2]\,dx = 0$$

$$= a_1 \left[-20a_1 \frac{x^2}{2} + 500\frac{x^2}{2} + 2a_1 \frac{x^3}{3} - 50\frac{x^3}{3}\right]_0^{10} = 0.$$

$$= \frac{-20a_1}{2}[10^2-0] + \frac{500}{2}[10^2-0] + \frac{2a_1}{3}[10^3-0] - \frac{50}{3}[10^3-0] = 0$$

$$= -1000a_1 + 25000 + 666.66a_1 - 16666.66 = 0$$

$$= -333.33a_1 = -8333.33$$

$$a_1 = 25 \rightarrow (8)$$

$$y = 25x(10-x)$$

Result:

Parameter, $a_1 = 25$.

10

ADDITIONAL SHEET

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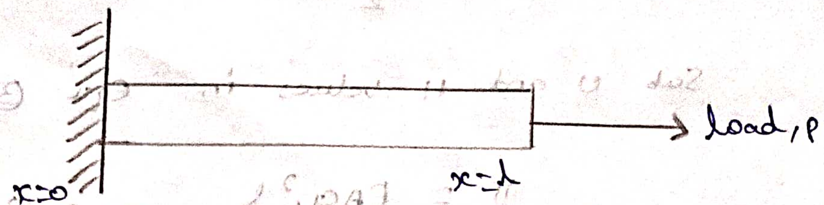

$$u = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots + a_n x^n$$

Case(i):

$$u = a_0 + a_1 x \rightarrow (1)$$

$$x = 0, u = 0$$

$$0 = a_0 + 0$$

$$a_0 = 0$$

sub a_0 value in equ (1)

$$u = a_1 x$$

$$du/dx = a_1 \rightarrow (2)$$

Total potential energy of bar; $\pi = U - H \rightarrow (3)$

$$U = \frac{EA}{2} \int_0^l \left(\frac{du}{dx}\right)^2 dx$$

$$= \frac{EA}{2} \int_0^l (a_1)^2 dx$$

$$u = \frac{EA\, a_1^2 l}{2} \rightarrow (4)$$

$$H = \int_0^l P dx = \int_0^l \rho u A dx$$

$$= \rho A \int_0^l u dx = \rho A \int_0^l a_1 x dx$$

$$= \rho A a_1 \left[x^2/2 \right]_0^l = \frac{\rho A a_1}{2} [l^2]$$

$$H = \frac{\rho A a_1 l^2}{2} \rightarrow ⑤$$

Sub U and H values in equ ③,

$$\pi = \frac{EA a_1^2 l}{2} - \frac{\rho A a_1 l^2}{2}$$

$$d\pi / da_1 = 0$$

$$\frac{EA(2a_1)l}{2} - \frac{\rho A l^2}{2} = 0$$

$$a_1 = \frac{\rho l}{2E}$$

Sub a_1 value in equ ②,

$$u = a_1 x = \rho l/2E \times x$$

$$u = \rho l/2E \times x$$

$$\delta u = u_l - u_0 = \rho l/2E \times l - 0.$$

$$= \frac{\rho l^2}{2E}$$

$$\delta u = \rho l^2/2E \rightarrow ⑥.$$

$$\sigma = E\, du/dx = E \times \rho l/2E$$

$$\sigma = \rho l/2 \rightarrow ⑦.$$

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Case (ii) :

$$u = a_0 + a_1 x + a_2 x^2 \rightarrow (8)$$

$$x = 0, \quad u = 0$$

$$0 = a_0 + 0 + 0$$

$$a_0 = 0$$

Sub a_0 value in equ (8),

$$u = a_1 x + a_2 x^2 \rightarrow (9)$$

$$du/dx = a_1 + 2a_2 x$$

$$\pi = U - H \rightarrow (10)$$

$$U = \frac{EA}{2} \int_0^l \left(\frac{du}{dx}\right)^2 dx = \frac{EA}{2} \int_0^l (a_1 + 2a_2 x)^2 dx$$

$$= \frac{EA}{2} \int_0^l \left[a_1^2 + (2a_2 x)^2 + 2a_1 2a_2 x\right] dx.$$

$$= \frac{EA}{2} \left[a_1^2 (l - 0) + \frac{4a_2^2}{3}(l^3 - 0) + \frac{4a_1 a_2}{2}(l^2 - 0)\right].$$

$$U = \frac{EA}{2}\left[a_1^2 l + \frac{4a_2^2}{3}(l^3) + 2a_1 a_2 (l^2)\right] \rightarrow (11).$$

$$H = \int_0^l p\,dx = \int_0^l \rho u A\,dx$$

$$= \rho A \int_0^l \left[a_1 x\,dx + a_2 x^2 dx\right]$$

$$H = \rho A \left[\frac{a_1}{2} l^2 + \frac{a_2}{3} l^3\right] \rightarrow (12).$$

Sub (11) and (12) values in (10).

$$\pi = U - H$$

$$\pi = \frac{EA}{2}\left[a_1^2 l + \frac{4a_2^2}{3}(l^3) + 2a_1 a_2 (l^2)\right] - eA\left[\frac{a_1}{2} l^2 + \frac{a_2}{3} l^3\right] \rightarrow (13).$$

$$\partial\pi/\partial a_1 = \frac{EA}{2}\left[2a_1 l + 0 + 2a_2 l^2\right] - eA\left[\frac{l^2}{2} + 0\right] = 0.$$

$$EA\left[a_1 l + a_2 l^2\right] = \frac{eAl^2}{2}$$

$$a_1 + a_2 l = el/2E \rightarrow (14).$$

$$\frac{EA}{2}\left[0 + \frac{8a_2}{3} l^3 + 2a_1 l^2\right] - eA\left[0 + l^3/3\right] = 0$$

$$= 8/3\, a_2 l^3 + 2a_1 l^2 = \frac{2el^3}{3E}$$

$$= 4/3\, a_2 l + a_1 = el/3E$$

$$a_1 = 1.333\, a_2 l = el/3E \rightarrow (15)$$

$$a_1 + a_2 l = el/2E$$

$$-0.333\, a_2 l = el/E\,(1/2 - 1/3)$$

$$= el/E\left(\frac{3-2}{6}\right)$$

$$-0.333\, a_2 = el/6E$$

$$a_2 = -e/2E$$

Sub a_2 value in equ (14)

$$a_1 + \left(\frac{-e}{2E} l\right) = el/2E$$

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$$a_1 = \frac{el}{2E} + \frac{el}{2E}$$

$$a_1 = \frac{2el}{2E}$$

$$a_1 = \frac{el}{E}$$

$$u = a_1 x + a_2 x^2$$

$$u = e/E \left[lx - x^2/2\right] \rightarrow (16)$$

$$\cancel{u_1 = e/E \times l^2/2}$$

$$\delta u = u_1 - u_0 = el^2/2E - 0$$

$$= el^2/2E \rightarrow (17)$$

$$u = e/E \left(lx - x^2/2\right)$$

$$\sigma = E\, du/dx = E \times e/E\,(l-x) = e(l-x)$$

$$\sigma = e(l-x) \rightarrow (18)$$

$$\delta l = \int_0^l \frac{P\,dx}{AE} = \int_0^l \frac{eAx}{AE}\,dx$$

$$\delta l = el^2/2E \rightarrow (19).$$

9

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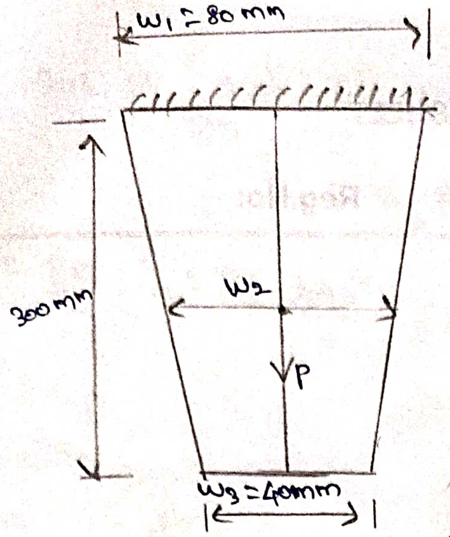

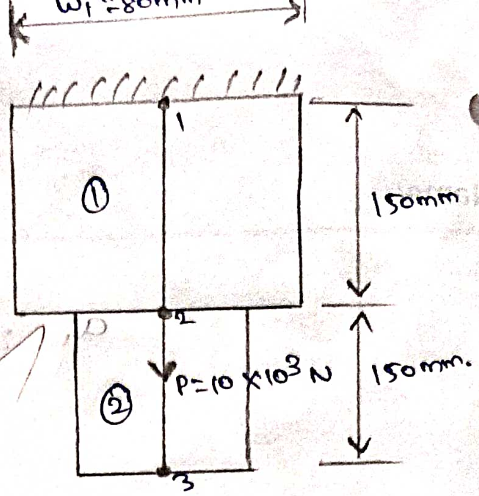

$A_1 = w_1 \times t_1$

$= 80 \times 10$

$A_1 = 800 mm^2$

$A_2 = w_2 \times t_2 = \left(\frac{w_1 + w_3}{2}\right) \times t_2$

$A_2 = 600 mm^2$

$A_3 = w_3 \times t_3 = 40 \times 10$

$A_3 = 400 mm^2$

$\overline{A_1} = \frac{800 + 600}{2}$

$\overline{A_1} = 700 mm^2$

$\overline{A_2} = \frac{600 + 400}{2}$

$\overline{A_2} = 500 mm^2$

Mass density, $\rho = 7800$ kg/m3

$= 7.6518 \times 10^{-5}$ N/mm3

weight density, $\rho = 7800 \times 9.81$ N/m3

$= 7.6518 \times 10^{-5}$ N/mm3

Young's modulus, $E = 2 \times 10^5$ MN/m2

$E = 2 \times 10^5 \times 10^6 \times 10^{-6}$ N/mm2

$P = 10 \times 10^3$ N.

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Sol:-

i) Force vector $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{\rho_1 A_1 l_1}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

$= \frac{7.6518 \times 10^{-5} \times 700 \times 150}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{Bmatrix} 4.017 \\ 4.017 \end{Bmatrix} \rightarrow (1)$

ii) force vector $\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \left(\frac{\rho_2 A_2 l_2}{2} \right) \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

$= \frac{7.6518 \times 10^{-5} \times 500 \times 150}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

$= 2.869 \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

$= \begin{Bmatrix} 2.869 \\ 2.869 \end{Bmatrix} \rightarrow (2)$.

$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 4.017 \\ 4.017 + 2.869 \\ 2.869 \end{Bmatrix} = \begin{Bmatrix} 4.017 \\ 6.886 \\ 2.869 \end{Bmatrix}$

$= \begin{Bmatrix} 4.017 \\ 10006.886 \\ 2.869 \end{Bmatrix} \rightarrow (3)$.

i) finite element equ its,

$\frac{A_1 E}{l_1} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$

$4.666 \times 2 \times 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$

$$2 \times 10^5 \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 4.666 & -4.666 \\ -4.666 & 4.666 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \rightarrow (4).$$

ii) finite element equ,

$$\frac{A_2 E_2}{l_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix}=\begin{Bmatrix}F_2\\ F_3\end{Bmatrix}$$

$$\frac{500\times 2\times10^5}{150}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix}=\begin{Bmatrix}F_2\\ F_3\end{Bmatrix}$$

$$2\times10^5\begin{bmatrix}3.333 & -3.333\\ -3.333 & 3.333\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix}=\begin{Bmatrix}F_2\\ F_3\end{Bmatrix} \rightarrow (5)$$

Assemble the equ (4) and (5),

$$\Rightarrow 2\times10^5\begin{bmatrix}4.666 & -4.666 & 0\\ -4.666 & 7.999 & -3.333\\ 0 & -3.333 & 3.333\end{bmatrix}\begin{Bmatrix}0\\ u_1\\ u_2\end{Bmatrix}=\begin{Bmatrix}4.017\\ 10006.886\\ 2.869\end{Bmatrix}$$

$$2\times10^5\begin{bmatrix}7.999 & -3.333\\ -3.333 & 3.333\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix}=\begin{bmatrix}10006.886\\ 2.869\end{bmatrix}$$

$$2\times10^5(7.999u_2-3.333u_3)=10006.886 \rightarrow (7)$$

$$2\times10^5(-3.333u_2+3.333u_3)=2.869 \rightarrow (8).$$

$$2\times10^5(4.666u_2)=10009.755$$

$$\therefore u_2=0.01073\,mm.$$

Sub u_2 value in equ (8),

$$2\times10^5[-3.333(0.01073)+3.333u_3]=2.869$$

$$u_3=0.01073\,mm$$

reaction force $(R)=[K](u)-(F)$

$$\begin{Bmatrix}R_1\\ R_2\\ R_3\end{Bmatrix}=2\times10^5\begin{bmatrix}4.666 & -4.666 & 0\\ -4.666 & 7.999 & -3.333\\ 0 & -3.333 & 3.333\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\\ u_3\end{Bmatrix}-\begin{Bmatrix}F_1\\ F_2\\ F_3\end{Bmatrix}$$

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$$= 2\times10^5 \begin{Bmatrix} -0.050 \\ 0.050 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 4.017 \\ 10006.886 \\ 2.869 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = \begin{Bmatrix} -10004.017 \\ -6.886 \\ -2.869 \end{Bmatrix}$$

$R_1 = -10004.017$ N

$R_2 = 6.886$ N

$R_3 = -2.869$ N

$R_1 + R_2 + R_3 = -10004.017 - 6.886 - 2.869$

$= -10013.772$ N

$F_1 + F_2 + F_3 = 4.017 + 10006.886 + 2.869$

$= 10013.772$ N.

9

PANDIAN SARASWATHI YADAV ENGINEERING COLLEGE

Madurai - Sivagangai Highway, Thirumansolai Post, Arasanoor, Sivagangai - 630 561

Read the instructions given below carefully before filling in the title page
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REGISTER NUMBER	9	1	2	0	2	0	1	1	4	0	0	1

Student Name	AJAY GOWTHAM.		
Degree / Branch	B.E - Mechanical Engineering		
Subject Code	ME 8692	Subject Title	FINITE ELEMENTS AND ANALYSIS.

Year / Semester / Sec	III / VI.
Date & Session	13-04-2023.FN
No. of Pages used	16.

All particulars given are verified

13/4/23

Name and Signature of the Hall Supdt. with date

PART - A		PART - B						GRAND TOTAL (IN WORDS)
Question No.	Marks	Question No.		Marks I	II	III	Total	
1	2	~~11~~ 8	a	9				
2	2		b					Eighty Six
3	2	~~12~~ 9	a	10				
4	2		b					
5	2	10 ~~13~~	a	10				
6	2		b					GRAND TOTAL
7	2	14	a	29				
8	14		b			43/70		
9		15	a					
10			b					86
Total								

Date 14/4/23	Signature of the Examiner	M. [signature] Signature of the HOD

Instructions to the Candidates :

1. Write your Register No. in the type as shown in the following example

6	2	0	1	1	3	7	5	1	0	2	1

2. Write your Register Number at the Top Right Hand Side of the QUESTION PAPER
3. Use both sides of the paper for answering questions.
4. Possession of any incriminating material and malpractice of any nature shall be punishable as per rules.
5. Answers must be legibly written in ink (Blue, Black or Blue Black)
6. Drawings and Sketches should be drawn using pencil.

1 types of loading acting on the structure

(i) Body force (f)

(ii) Traction force (T)

(iii) Point load

2. natural co-ordinates

A natural co-ordinates system is used to define any point inside the element by a set of dimensionless numbers whose magnitude never exceeds unity. This system is very useful in assembling of stiffness matrices

3. shape function

In finite element method field variables within an element are generally expressed by the following approximate relation

$$\phi(x,y) = N_1(x,y)\phi_1 + N_2(x,y)\phi_2 + N_3(x,y)\phi_3$$

where ϕ_1, ϕ_2 and ϕ_3 are the values of the field variable at the nodes and N_1, N_2 and N_3 are the interpolation function

N_1, N_2 and N_3 are also called shape functions because they are used to express the geometry or shape of the element

4 properties of a stiffness matrix

1. it is symmetric matrix

2. The sum of elements in any column must be equal to zero

3. It is an unstable element so the ... is equal to zero

5 CST element

strain displacement matrix for CST element is

$$[B] = \frac{1}{2A}\begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

A = Area of the element

$q_1 = y_2 - y_3$; $q_2 = y_3 - y_1$; $q_3 = y_1 - y_2$

$r_1 = x_3 - x_2$; $r_2 = x_1 - x_3$; $r_3 = x_2 - x_1$

6 Theory of pure torsion

1. twist is uniform along the length of the shaft

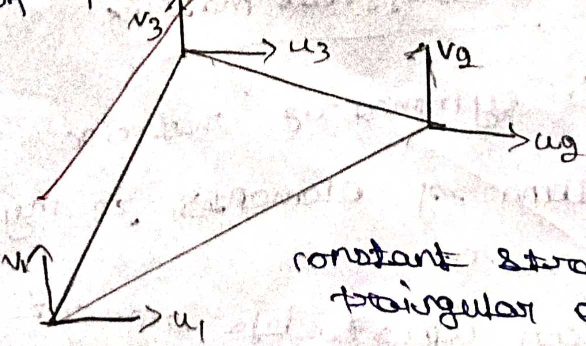
2. The stress does not exceed the limit of proportionality

3. strain and deformation are small

7 CST element

1) Three Noded triangular element is known constant strain triangle

ii) It has six unknown displacement degrees of freedom



constant strain triangular element

~~merit~~ merit

1. calculation of stiffness matrix is easier

Demerit

2. The result will be poor

1

$d = 2cm = 0.02m$

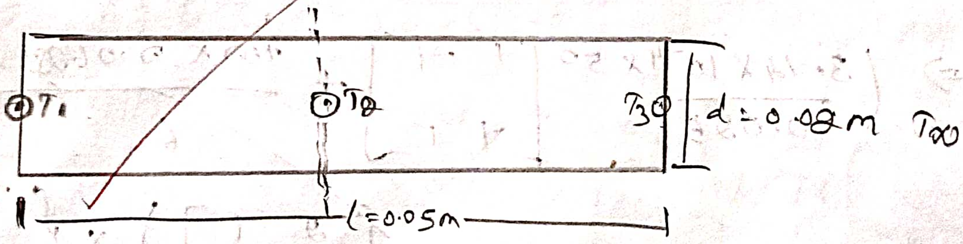
$l = 5cm = 0.05m$

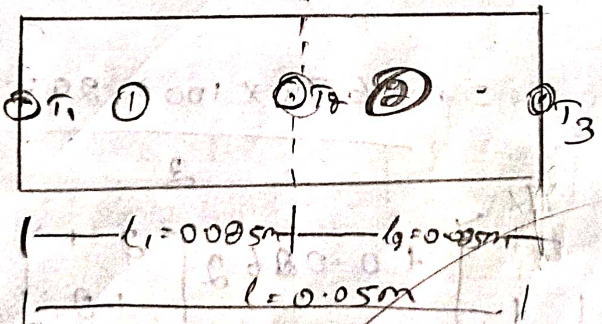
$k = 50 W/m^\circ c$

$T_1 = 320^\circ c + 273 = 593 K$

$T_\infty = 20^\circ c + 273 = 293 K$

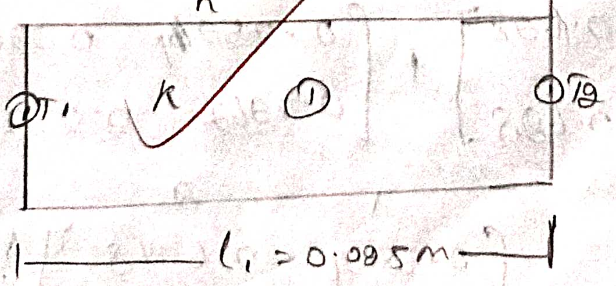
$h = 100 W/m^2 \,^\circ c$





For element (nodes 1, 2)

h



$$\left(\frac{AK}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + \frac{hPl_1}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\right)\begin{Bmatrix}T_1\\ T_2\end{Bmatrix} = \frac{QAl_1 + PhT_\infty l_1}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix}$$

where P = perimeter $= \pi d = \pi \times 0.02$

$P = 0.0628\ m$

A = area $= \frac{\pi}{4} d^2$

$\frac{\pi}{4}(0.02)^2$

$A = 3.14 \times 10^{-4} m^2$

Heat generation Q is not given so neglect that terms $\left(\frac{QAl_1}{2}\right)$

$$\left(\frac{AK}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + \frac{hPl_1}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\right)\begin{Bmatrix}T_1\\ T_2\end{Bmatrix} = \frac{PhT_\infty l_1}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix}$$

$$\Rightarrow \left(\frac{3.14\times10^{-4}\times50}{0.025}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + \frac{100\times0.0628\times0.025}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\right)\times\begin{Bmatrix}T_1\\ T_2\end{Bmatrix}$$

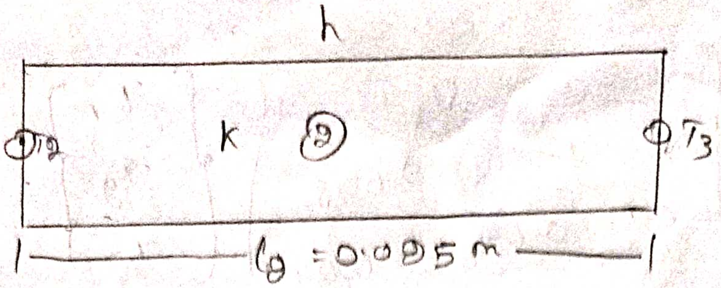
$$= \frac{0.0628\times100\times293\times0.025}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix}$$

$$\left(0.628\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + 0.0262\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\right)\begin{Bmatrix}T_1\\ T_2\end{Bmatrix} = 23\begin{Bmatrix}1\\ 1\end{Bmatrix}$$

$$\left(\begin{bmatrix}0.628 & -0.628\\ -0.628 & 0.628\end{bmatrix} + \begin{bmatrix}0.0524 & 0.0262\\ 0.0262 & 0.0524\end{bmatrix}\right)\begin{Bmatrix}T_1\\ T_2\end{Bmatrix} = \begin{Bmatrix}23\\ 23\end{Bmatrix}$$

$$\begin{array}{c c} & \begin{array}{cc}1 & 2\end{array}\\ \begin{array}{c}1\\ 2\end{array} & \begin{bmatrix}0.6804 & -0.6018\\ -0.6018 & 0.6804\end{bmatrix}\end{array}\begin{Bmatrix}T_1\\ T_2\end{Bmatrix} = \begin{Bmatrix}23\\ 23\end{Bmatrix}\begin{array}{c}1\\ 2\end{array}$$

For element 2 (Nodes 2, 3)



$$\begin{array}{cc} \begin{array}{cc} 2 & 3 \end{array} & \\ \begin{bmatrix} 0.6804 & -0.6018 \\ -0.6018 & 0.6804 \end{bmatrix} & \begin{array}{c} 2 \\ 3 \end{array} \end{array} \begin{Bmatrix} T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 23 \\ 23 \end{Bmatrix} \begin{array}{c} 2 \\ 3 \end{array}$$

$$\begin{bmatrix} 0.6804 & -0.6804 & 0 \\ -0.6018 & 0.6804 + 0.6804 & -0.6018 \\ 0 & -0.6018 & 0.6804 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 23 \\ 23 + 23 \\ 23 \end{Bmatrix}$$

$$\begin{bmatrix} 0.6804 & -0.6018 & 0 \\ -0.6018 & 1.3608 & -0.6018 \\ 0 & -0.6018 & 0.6804 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 23 \\ 46 \\ 23 \end{Bmatrix}$$

Step 1

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.3608 & -0.6018 \\ 0 & -0.6018 & 0.6804 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 23 \\ 46 \\ 23 \end{Bmatrix}$$

Step 2

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.3608 & -0.6018 \\ 0 & -0.6018 & 0.6804 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 593 \\ 46 \\ 23 \end{Bmatrix}$$

Step 3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.3608 & -0.6018 \\ 0 & -0.6018 & 0.6804 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 593 \\ 46+356.867 \\ 23 \end{Bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.3608 & -0.6018 \\ 0 & -0.6018 & 0.680 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} 593 \\ 402.867 \\ 23 \end{Bmatrix}$$

$$1.3608 T_2 - 0.6018 T_3 = 402.867$$
$$-0.6018 T_2 + 0.6804 T_3 = 23$$

$$1.5385 T_2 - 0.6804 T_3 = 455.485$$
$$-0.6018 T_2 + 0.6804 T_3 = 23$$

$$0.9367 T_2 = 478.485$$

$$\boxed{T_2 = 510.819\,K}$$

Result

Temperature at the mid point of the rod $T_2 = 510.819\,K$

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ADDITIONAL SHEET

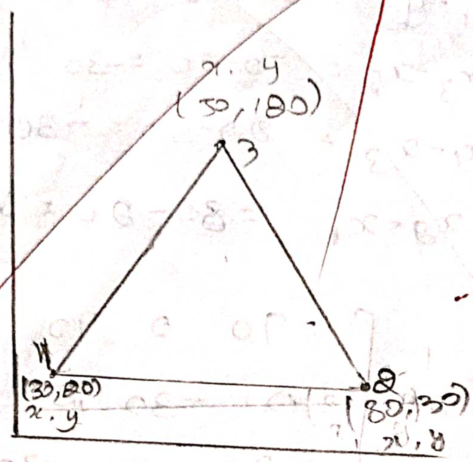
Name: Reg.No:

Q. Given

$x_1 = 20mm$ $y_1 = 30mm$

$x_2 = 80mm$ $y_2 = 30mm$

$x_3 = 50mm$ $y_3 = 120mm$



Young's modulus $E = 210GPa \Rightarrow 210 \times 10^3 N/mm^2$

$E = 2.1 \times 10^5 N/mm^2$

Poisson's ratio $\nu = 0.25$

Thickness $t = 10mm$

To find stiffness matrix [K]

Stiffness matrix $[K] = [B]^T [D] [B] At$

where A - Area of the element

$$= \frac{1}{2} \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$$

$$= 1/2 \begin{bmatrix} 1 & 20 & 30 \\ 1 & 80 & 30 \\ 1 & 50 & 120 \end{bmatrix}$$

$= 1/2 \times [1 \times (80 \times 120 - 50 \times 30) - 20(120 - 30) + 30(50 - 80)]$

$= 1/2 \times [8100 - 1800 - 900]$

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$A = 2700 mm^2$

Strain displacement matrix $[B] = \frac{1}{2A}\begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$

$q_1 = y_2 - y_3 = 30 - 120 = -90$

$q_2 = y_3 - y_1 = 120 - 30 = 90$

$q_3 = y_1 - y_2 = 30 - 30 = 0$

$r_1 = x_3 - x_2 = 50 - 80 = -30$

$r_2 = x_1 - x_3 = 20 - 50 = -30$

$r_3 = x_2 - x_1 = 80 - 20 = 60$

$$[B] = \frac{1}{2A}\begin{bmatrix} -90 & 0 & 90 & 0 & 0 & 0 \\ 0 & -30 & 0 & -30 & 0 & 60 \\ -30 & -90 & -30 & 90 & 60 & 0 \end{bmatrix}$$

$$= \frac{1}{2 \times 2700}\begin{bmatrix} -90 & 0 & 90 & 0 & 0 & 0 \\ 0 & -30 & 0 & -30 & 0 & 60 \\ -30 & -90 & -30 & 90 & 60 & 0 \end{bmatrix}$$

$$[B] = 5.555 \times 10^{-3}\begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

$$[D] = \frac{E}{1-V^2}\begin{bmatrix} 1 & V & 0 \\ V & 1 & 0 \\ 0 & 0 & \frac{1-V}{2} \end{bmatrix}$$

$$= \frac{2.1 \times 10^5}{1-(0.25)^2}\begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1-0.25}{2} \end{bmatrix}$$

$$= \frac{2.1\times10^5\times0.25}{0.9375}\begin{bmatrix}4 & 1 & 0\\ 1 & 4 & 0\\ 0 & 0 & 1.5\end{bmatrix}$$

$$[D] = 56\times10^3\begin{bmatrix}4 & 1 & 0\\ 1 & 4 & 0\\ 0 & 0 & 1.5\end{bmatrix}$$

$$[D][B] = 56\times10^3\begin{bmatrix}4 & 1 & 0\\ 1 & 4 & 0\\ 0 & 0 & 1.5\end{bmatrix}\times 5.555\times10^{-3}$$

$$\begin{bmatrix}-3 & 0 & 3 & 0 & 0 & 0\\ 0 & -1 & 0 & -1 & 0 & 2\\ -1 & -3 & -1 & 3 & 2 & 0\end{bmatrix}$$

$$= 311.08\times\begin{bmatrix}4 & 1 & 0\\ 1 & 4 & 0\\ 0 & 0 & 1.5\end{bmatrix}\begin{bmatrix}-3 & 0 & 3 & 0 & 0 & 0\\ 0 & -1 & 0 & -1 & 0 & 2\\ -1 & -3 & -1 & 3 & 2 & 0\end{bmatrix}$$

$$= 311.08\begin{bmatrix}-12+0+0 & 0-1+0 & 12+0+0 & 0-1+0 & 0+0+0 & 0+2+0\\ -3+0+0 & 0-4+0 & 3+0+0 & 0-4+0 & 0+0+0 & 0+8+0\\ 0+0-1.5 & 0+0-4.5 & 0+0-1.5 & 0+0+4.5 & 0+0+3 & 0+0+0\end{bmatrix}$$

$$= 311.08\begin{bmatrix}-12 & -1 & 12 & -1 & 0 & 2\\ -3 & -4 & 3 & -4 & 0 & 8\\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0\end{bmatrix}$$

$$[B] = 5.555\times10^{-3}\begin{bmatrix}-3 & 0 & 3 & 0 & 0 & 0\\ 0 & -1 & 0 & -1 & 0 & 2\\ -1 & -3 & -1 & 3 & 2 & 0\end{bmatrix}$$

$$[B]^T = 5.555\times10^{-3}\begin{bmatrix}-3 & 0 & -1\\ 0 & -1 & -3\\ 3 & 0 & -1\\ 0 & -1 & 3\\ 0 & 0 & 2\\ 0 & 2 & 0\end{bmatrix}$$

ADDITIONAL SHEET

Name: Reg.No:

$$[B]^T[D][B] = 5.555 \times 10^{-3} \begin{bmatrix} -3 & 0 & 1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \times 311.08$$

$$\begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}$$

$$[B]^T[D][B] = 1.798 \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 6 & 16 \end{bmatrix}$$

$\times$ 2700 × 10 N/m

$$[k] = 46.656 \times 103 \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix}$$

ADDITIONAL SHEET

Name: Reg.No:

10 Given data

$t = 25mm$

$E = 2 \times 10^5 N/mm^2$

$V = 0.30$

$b = 250mm$

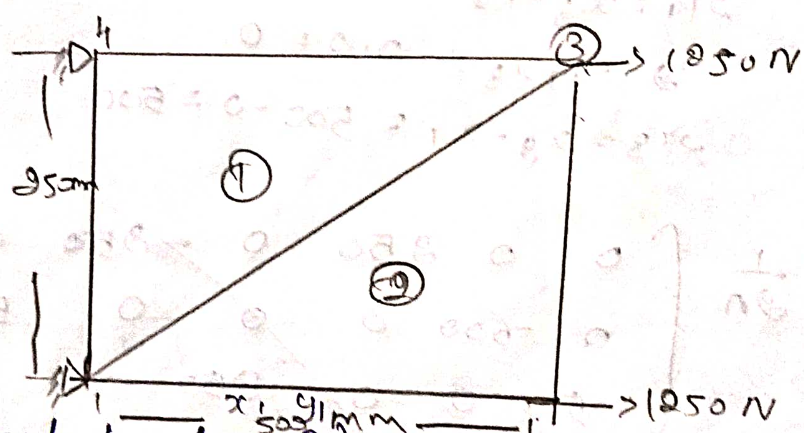
$l = 500mm$

$T = 0.4 N/mm^2$

$E = \frac{1}{2} TA = \frac{1}{2} \times T \times (b \times t)$

$= \frac{1}{2} \times 0.4 \times 250 \times 25$

$\boxed{F = 1250N}$



For node 1: $(\overset{x_1}{0}, \overset{y_1}{0})$

For node 3: $(\overset{x_2}{500}, \overset{y_2}{250})$

For node 4: $(\overset{x_3}{0}, \overset{y_3}{250})$

Stiffness matrix $[K]_1 = [B]^T [D] [B] At$

where A = Area of the triangular element

$\frac{1}{2} \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 500 & 250 \\ 1 & 0 & 250 \end{bmatrix}$$

$$= 1/2 \times 1(500 \times 250 - 0)$$

$$A = 62500\,mm^2 = 62.5 \times 10^3\,mm^2$$

$$\boxed{A = 62.5 \times 10^3\,mm^2}$$

$$[B] = \frac{1}{2A}\begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

$q_1 = y_2 - y_3 = 250 - 250 = 0$

$q_2 = y_3 - y_1 = 250 - 0 = 250$

$q_3 = y_1 - y_2 = 0 - 250 = -250$

$r_1 = x_3 - x_2 = 0 - 500 = -500$

$r_2 = x_1 - x_3 = 0 - 0 = 0$

$r_3 = x_2 - x_1 = 500 - 0 = 500$

$$B = \frac{1}{2A}\begin{bmatrix} 0 & 0 & 250 & 0 & -250 & 0 \\ 0 & -500 & 0 & 0 & 0 & 500 \\ -500 & 0 & 0 & 250 & 500 & -250 \end{bmatrix}$$

$$= \frac{250}{2 \times 62.5 \times 10^3}\begin{bmatrix} 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & 0 & 0 & 1 & 2 & -1 \end{bmatrix}$$

$$[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

ADDITIONAL SHEET

Name: Reg.No:

$$= \frac{2\times10^5}{1-(0.3)^2}\begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & \frac{1-0.3}{2} \end{bmatrix}$$

$$[D] = \frac{2\times10^5}{0.91}\begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 0.35 \end{bmatrix}$$

$$[B][D] = \frac{2\times10^5}{0.91}\begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 0.35 \end{bmatrix} \times \frac{250}{2\times62.5\times10^3}\begin{bmatrix} 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 2 \\ -2 & 0 & 0 & 1 & 2 & -1 \end{bmatrix}$$

$$= 439.56\begin{bmatrix} 0 & -0.6 & 1 & 0 & -1 & 0.6 \\ 0 & -2 & 0.3 & 0 & -0.3 & 2 \\ -0.7 & 0 & 0 & 0.35 & 0.7 & -0.35 \end{bmatrix}$$

$$[B] = \frac{250}{2\times62.5\times10^3}\begin{bmatrix} 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 \\ -2 & 0 & 0 & 1 & 2 & -1 \end{bmatrix}$$

$$B^T = 2\times10^3\begin{bmatrix} 0 & 0 & -2 \\ 0 & -2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 2 \\ 0 & 2 & -1 \end{bmatrix}$$

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$[B]^T [D] [B]$ ~~...~~ and A, t values in equation (1)

Stiffness matrix $[K]_1 = 0.8791 \begin{bmatrix} 1.4 & 0 & 0 & -0.7 & -1.4 & 0.7 \\ 0 & 4 & -0.6 & 0 & 0.6 & -1 \\ 0 & -0.6 & 1 & 0 & -1 & 0.6 \\ -0.7 & 0 & 0 & 0.35 & 0.7 & -0.35 \\ -1.4 & 0.6 & -1 & 0.7 & 2.4 & -1.3 \\ 0.7 & -4 & 0.6 & -0.35 & -1.3 & 4.35 \end{bmatrix} \times 6.25 \times 10^3 \times 25$

$$[K_1] = 1373.59 \times 10^3 \begin{bmatrix} 1.4 & 0 & 0 & -0.7 & -1.4 & 0.7 \\ 0 & 4 & -0.6 & 0 & 0.6 & -4 \\ 0 & -0.6 & 1 & 0 & -1 & 0.6 \\ -0.7 & 0 & 0 & 0.35 & 0.7 & -0.35 \\ -1.4 & 0.6 & -1 & 0.7 & 2.4 & -1.3 \\ 0.7 & -4 & 0.6 & -0.35 & -1.35 & 4.35 \end{bmatrix}$$

$$[K_1] = 1 \times 10^3 \begin{array}{c} \begin{array}{cccccc} u_1 & v_1 & u_3 & v_3 & u_4 & v_4 \end{array} \\ \begin{bmatrix} 1923.026 & 0 & 0 & -961.513 & -1923.026 & 961.513 \\ 0 & 5494.36 & 884.154 & 0 & 884.154 & -5494.36 \\ 0 & -884.154 & 1373.59 & 0 & -1373.59 & 884.154 \\ -961.513 & 0 & 0 & 480.7585 & 961.513 & -480.7585 \\ -1923.026 & 884.154 & -1373.59 & 961.513 & 3296.616 & -1785.667 \\ 961.513 & -5494.36 & 884.154 & -480.785 & -1785.55 & 5745.1165 \end{bmatrix} \end{array}$$

For element 2

Take node 1 as origin

For node 1 : $(x_1, y_1) = (0, 0)$

For node 2 : $(x_2, y_2) = (500, 0)$

For node 3 : $(x_3, y_3) = (500, 250)$

ADDITIONAL SHEET

Name: Reg.No:

$$\frac{1}{2}\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 500 & 0 \\ 1 & 500 & 250 \end{bmatrix}$$

$$\boxed{A = 62.5 \times 10^3\ mm^2}$$

$$[B] = \frac{1}{2A}\begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

$$[B] = \frac{1}{2A}\begin{bmatrix} -250 & 0 & 250 & 0 & 0 & 0 \\ 0 & 0 & 0 & -500 & 0 & 500 \\ 0 & -250 & -500 & 250 & 500 & 0 \end{bmatrix}$$

$$[B] = \frac{250}{2 \times 62.5 \times 10^3}\begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 2 \\ 0 & -1 & -2 & 1 & 2 & 0 \end{bmatrix}$$

$$[B]^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & -2 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

$[D][B]$

$$= 439.56\begin{bmatrix} -1 & 0 & 1 & -0.6 & 0 & 0.6 \\ -0.3 & 0 & 0.3 & -2 & 0 & 2 \\ 0 & -0.35 & 0.7 & 0.35 & 0.7 & 0 \end{bmatrix}$$

~~Select~~ For element (2) nodal displacements are $u_1\ v_1\ u_2\ v_2\ u_3\ v_3$ [Refer Fig 1 v]

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$$K_2 = 1\times10^3 \begin{array}{cccccc} u_1 & v_1 & u_2 & v_2 & u_3 & v_3 \end{array}$$

$$\begin{bmatrix} 1373.59 & 0 & -1373.59 & 824.154 & 0 & -824.154 \\ 0 & 480.7565 & 961.513 & -480.7565 & -961.513 & 0 \\ -1373.59 & 961.513 & 3296.616 & -1785.667 & -1923.096 & 824.154 \\ 824.154 & -480.7565 & -1785.667 & 5975.1165 & 961.513 & -5494.36 \\ 0 & -961.513 & -1923.096 & 961.513 & 1923.096 & 0 \\ -824.154 & 0 & 824.154 & -5494.36 & 0 & 5494.36 \end{bmatrix}$$

Result:-

Global Stiffness matrix $[K] = 10^3 \times$

$$\begin{bmatrix} 3296.616 & 0 & -1373.59 & 824.154 & 0 & -1785.667 & -1923.096 & 961.513 \\ 0 & 5675.1165 & 961.513 & -480.7565 & \cancel{-1785.667}\ 0 & 824.154 & -5494.36 & \\ 824.154 & -480.7565 & -1785.667 & 5975.1165 & 961.513 & -5494.36 & 0 & 0 \\ 0 & -1785.667 & -1923.096 & 961.513 & 3296.616 & 0 & -1373.59 & 824.154 \\ -1785.667 & 0 & 824.154 & -5494.36 & 0 & 5975.1165 & 961.513 & -480.7565 \\ -1923.096 & 824.154 & 0 & 0 & -1373.59 & 961.513 & 3296.616 & -1785.667 \\ 961.513 & -5494.36 & 0 & 0 & 824.154 & -480.7565 & -1785.667 & 5975.1165 \end{bmatrix}$$

PANDIAN SARASWATHI YADAV ENGINEERING COLLEGE

Madurai - Sivagangai Highway, Thirumansolai Post, Arasanoor, Sivagangai - 630 561

Read the instructions given below carefully before filling in the title page
(To be filled in by the candidate)

REGISTER NUMBER	9	1	2	0	2	0	1	1	4	0	0	1

Student Name	Ajay crowtham		
Degree / Branch	B.E. Mechanical Engineering		
Subject Code	ME 8692	Subject Title	FINITE ELEMENTS AND ANALYSIS.

Year / Semester / Sec	III / VI.
Date & Session	18-05-2023 AN.
No. of Pages used	27

All particulars given are verified

18/5/23.

Name and Signature of the Hall Supdt. with date

PART - A		PART - B						GRAND TOTAL (IN WORDS)
Question No.	Marks	Question No.		Marks I	II	III	Total	
1	2	11	a	11				Eighty Three
2	2		b					
3	2	12	a	11				
4	2		b					
5	2	13	a	10.				
6	2		b			83/100		GRAND TOTAL
7	2	14	a	10				
8	2		b					
9	2	15	a	9.				83%.
10	2		b					
Total	20	16 ~~+~~		12				

18/05/23 Date	Signature of the Examiner 63	Signature of the HOD

Instructions to the Candidates :

1. Write your Register No. in the type as shown in the following example

6	2	0	1	1	3	7	5	1	0	2	1

2. Write your Register Number at the Top Right Hand Side of the QUESTION PAPER
3. Use both sides of the paper for answering questions.
4. Possession of any incriminating material and malpractice of any nature shall be punishable as per rules.
5. Answers must be legibly written in ink (Blue, Black or Blue Black)
6. Drawings and Sketches should be drawn using pencil.

PART- A

1. The Polynomial type of interpolation functions are mostly used due to the following reasons:

1. It is easy to formulate and computerize the finite element equations.

2. It is easy to perform differentiation or integration

3. The accuracy of the results can be improved by increasing the order of the polynomial.

2. The art of subdividing a structure into a convenient number of smaller components is known as discretization. These smaller components are then put together. The process of uniting the various elements together is called assemblage.

3. $[k]\{u\} = \omega^2 (m)(u)$ $\Rightarrow$ k - stiffness matrix

u - displacement

ω - Natural frequency

m - mass matrix

$$k = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

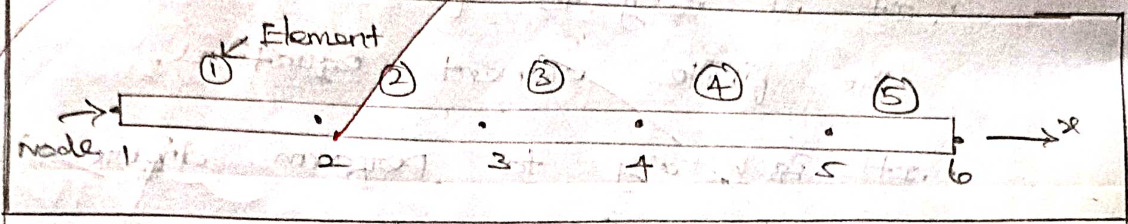
$$m = \frac{\rho AL}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$m = \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

4. Global co-ordinates:

The Points in the entire structure are defined using co-ordinate system is known as global co-ordinate system.

ex:



natural co-ordinates:

A natural co-ordinate system is used to defined any Point inside the element by a set of dimensionless numbers. whose magnitude never exceds never exceeds unity. The system is very useful in assembling of stiffness matrices.

5.

* The material of the shaft is homogeneous, Perfectly elastic and obeys Hooke's law.

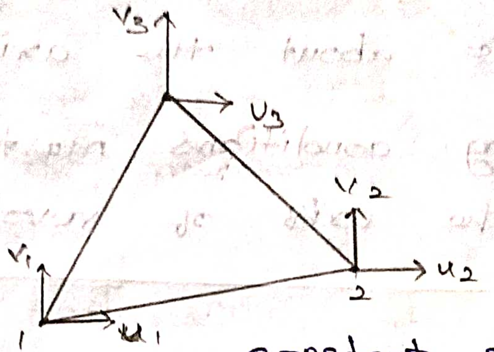
* Twist is uniform along the length of the shaft.

* The stress does not exceed the limit of Proportionality.

* strain and deformation are small.

6. **CST:**

Three noded triangular element is known constant strain Triangle, which is it has six unknown displacement degrees of freedom.



constant strain triangular element

merit:

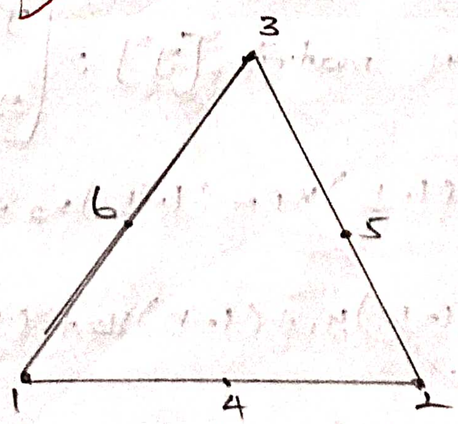
calculation of stiffness matrix is easier.

Demerite:

The strain variation within the element is considered as constant so, the result will be poor.

LST:

six noded triangular element is known as linear strain Triangular. It has twelve unknown displacement degrees of freedom.



7. * The problem domain must be symmentric about the axis of revolution.

* All boundary conditions must be symmentric about the axis of revolution

* All loading conditions must be symmentric about the axis of revolution.

8.

$$B = \frac{1}{2A}\begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ \frac{\alpha_1}{r}+\beta_1+\frac{\gamma_1 z}{r} & 0 & \frac{\alpha_2}{r}+\beta_2+\frac{\gamma_2 z}{r} & 0 & \frac{\alpha_3}{r}+\beta_3+\frac{\gamma_3 z}{r} & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 \end{bmatrix}$$

co-ordinate, $r = \frac{r_1+r_2+r_3}{3}$

$z = \frac{z_1+z_2+z_3}{3}$

$\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3 \rightarrow$ Refer questi

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9. Jacobian matrix, $[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$

$J_{11} = \frac{1}{4}\left[-(1-\eta)x_1-(1-\eta)x_2+(1+\eta)x_3-(1+\eta)x_4\right]$

$J_{12} = \frac{1}{4}\left[-(1-\eta)y_1+(1-\eta)y_2+(1+\eta)y_3-(1+\eta)y_4\right]$

$J_{21} = \frac{1}{4}\left[-(1-\varepsilon)x_1-(1+\varepsilon)x_2+(1+\varepsilon)x_3+(1-\varepsilon)x_4\right]$

$J_{22} = \frac{1}{4}\left[-(1-\varepsilon)y_1-(1+\varepsilon)y_2+(1+\varepsilon)y_3+(1-\varepsilon)y_4\right]$

10. Resonance:

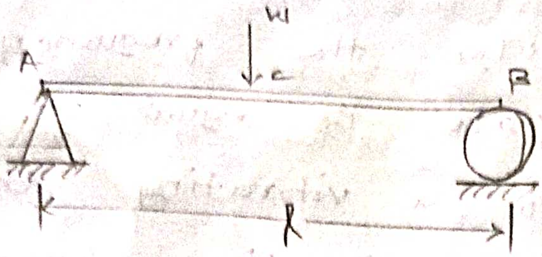
when the frequency of external force is equal to the natural frequency of a vibrating body, the amplitude of vibration becomes excessively large. This phenomenon is known as resonance.

Dynamic Analysis:

when the inertia effect due to the mass of the components is also considered in addition to the externally applied load, then the analysis is called dynamic analysis.

PART- B

11. (b)



To find:

Deflection at midspan, y_{max}.

Soln:

$$Y = a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{3\pi x}{l}$$

The potential energy of beam is given by,

$$\pi = U - H$$

$U \rightarrow$ strain energy

$H \rightarrow$ workdone by external force.

$$U = \frac{EI\pi^4}{4l^3}\left[a_1^2 + 81a_2^2\right]$$

$$H = W y_{max}$$

$$y = a_1 \sin \frac{\pi x}{l} + a_2 \sin \frac{3\pi x}{l}$$

$$x = l/2$$

$$\Rightarrow y_{max} = a_1 \sin \frac{\pi l/2}{l} + a_2 \sin \frac{3\pi l/2}{l}$$

$$= a_1 \sin \pi/2 + a_2 \sin 3\pi/2$$

$$\boxed{Y_{max} = a_1 - a_2}$$

$$\Rightarrow H = W(a_1 - a_2)$$

$$\Rightarrow \pi = \frac{EI\pi^4}{4l^3}\left[a_1^2 + 81a_2^2\right] - W(a_1 - a_2)$$

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For stationary value of π, the following conditions must be satisfied.

$$\frac{\partial\pi}{\partial a_1} = 0 \quad \& \quad \frac{\partial\pi}{\partial a_2} = 0$$

$$\frac{\partial\pi}{\partial a_1} = \frac{EI\pi^4}{4l^3}[2a_1] - W = 0$$

$$\Rightarrow \frac{EI\pi^4}{4l^3}(2a_1) - W = 0$$

$$\frac{EI\pi^4}{2l^3}(a_1) = W$$

$$\boxed{a_1 = \frac{2l^3 W}{EI\pi^4}}$$

$$\frac{\partial\pi}{\partial a_2} = \frac{EI\pi^4}{4l^3}[162a_2] + W = 0$$

$$\frac{EI\pi^4}{4l^3}(162a_2) + W = 0$$

$$\frac{81EI\pi^4}{2l^3}a_2 = -W$$

$$\boxed{a_2 = \frac{-2l^3 W}{81EI\pi^4}}$$

Maximum deflection, $y_{max} = a_1 - a_2$

$$Y_{max} = \frac{2l^3 W}{EI\pi^4} + \frac{2l^3 W}{81EI\pi^4} = \frac{2l^3 W}{EI\pi^4}\left(1 + \frac{1}{81}\right)$$

$$= \frac{2l^3 w}{EI\pi^4}(1.0123)$$

$$= \frac{2.0246\, l^3 w}{EI\pi^4} = 0.0207\, \frac{wl^3}{EI}$$

$$\boxed{Y_{max} = \frac{wl^3}{48.1EI}} \quad \text{——— ①}$$

we know that simply supported beam subjected to point load at centre, maximum deflection is,

$$Y_{max} = \frac{wl^3}{48EI} \quad \text{——— ②}$$

From eqn ① and ②, we know that exact soln and solution obtained by using Rayleigh Ritz method are almost same. In order to get accurate result, more terms in fourier series should be taken.

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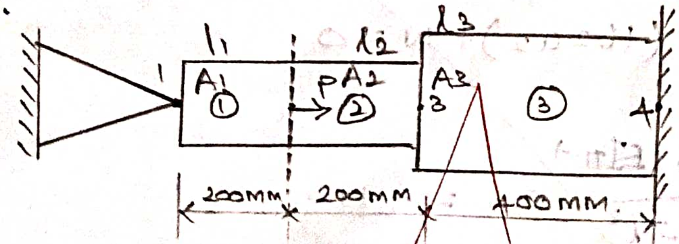
12. (a)

Given:



$A_1 = 300\,mm^2$

$A_2 = 300\,mm^2$

$A_3 = 600\,mm^2$

$l_1 = 200\,mm$

$l_2 = 200\,mm$

$l_3 = 400\,mm$

$E = 2\times10^5\,N/mm^2$, $P = 400\,kN = 400\times10^3\,N$

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Soln:

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{AE}{l}\begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

For element (1):

$$\frac{A_1E_1}{l_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$\frac{300 \times 2\times10^5}{200}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$3\times10^5\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$1\times10^5\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \end{array}\begin{array}{c} 1 \\ 2 \end{array}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

For element (2):

$$\frac{A_2E_2}{l_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}$$

$$\frac{300 \times 2\times10^5}{200}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}$$

$$= \frac{2l^3 w}{\pi t ...}(1.0123)$$

$$3\times10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}$$

$$1\times10^5 \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}$$

For element (3):

$$\frac{A_3 E_3}{l_3} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix}$$

$$\frac{600\times 2\times10^5}{400} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix}$$

$$3\times10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix}$$

$$1\times10^5 \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix}$$

Assemble the finite elements:

$$1\times10^5 \begin{bmatrix} -3 & -3 & 0 & 0 \\ -3 & 3+3 & -3 & 0 \\ 0 & -3 & 3+3 & -3 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

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e: Reg.No:

$$1\times10^5 \begin{bmatrix} 3 & -3 & 0 & 0 \\ -3 & 6 & -3 & 0 \\ 0 & -3 & 6 & -3 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

Boundary conditions:

(i) $u_1 = u_4 = 0$

(ii) $F_2 = 400\times10^3\,N$

(iii) $F_1 = F_3 = F_4 = 0$

$$1\times10^5 \begin{bmatrix} -3 & -3 & 0 & 0 \\ -3 & 6 & -3 & 0 \\ 0 & -3 & 6 & -3 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 400\times10^3 \\ 0 \\ 0 \end{Bmatrix}$$

$$1\times10^5 \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 400\times10^3 \\ 0 \end{Bmatrix}$$

$$1\times10^5\,(6u_2 - 3u_3) = 400\times10^3$$

$$1\times10^5\,(-3u_2 + 6u_3) = 0$$

$$1\times10^5\,(6u_2 - 3u_3) = 400\times10^3$$

$$1\times10^5\,(-6u_2 + 12u_3) = 0$$

$$1\times10^5\,(9u_3) = 400\times10^3$$

$$u_3 = 0.4444 \text{ mm}$$

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$$1\times10^5\left[6u_2-3(0.4444)\right] = 400\times10^3$$

$$\boxed{u_2 = 0.8888\,mm}$$

$$\sigma = E\,\frac{du}{dx}$$

$$\sigma_1 = E_1 \times \frac{u_2-u_1}{l_1}$$

$$= 2\times10^5 \times \frac{(0.8888-0)}{200}$$

$$\boxed{\sigma_1 = 888.88\ N/mm^2}$$

$$\sigma_2 = E_2 \times \frac{(u_3-u_2)}{l_2}$$

$$\boxed{\sigma_2 = -444.44\ N/mm^2}$$

$$\sigma_3 = E_3 \times \frac{(u_4-u_3)}{l_3}$$

$$\boxed{\sigma_3 = -222.22\ N/mm^2}$$

$$\{R\} = [K]\{u^*\} - \{F\}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{Bmatrix} = 1\times10^5 \begin{bmatrix} -3 & -3 & 0 & 0 \\ -3 & 6 & -3 & 0 \\ 0 & -3 & 6 & -3 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} - \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

$$= 1\times10^5 \begin{bmatrix} -3 & -3 & 0 & 0 \\ -3 & 6 & -3 & 0 \\ 0 & -3 & 6 & -3 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.8888 \\ 0.4444 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 400\times10^3 \\ 0 \\ 0 \end{Bmatrix}$$

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$$= 1\times10^5 \begin{Bmatrix} -2.6667 \\ 4 \\ 0 \\ -1.3333 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 400\times10^3 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{Bmatrix} = \begin{Bmatrix} -2.6667\times10^5 \\ 0 \\ 0 \\ -1.3333\times10^5 \end{Bmatrix}$$

$R_1 = -2.6667\times10^5 N$, $R_2 = 0$, $R_3 = 0$, $R_4 = -1.33\times10^5 N$

Result:

$u_1 = 0$

$u_2 = 0.8888$ mm

$u_3 = 0.4444$ mm

$u_4 = 0$

$\sigma_1 = 888.88 \ N/mm^2$

$\sigma_2 = -444.44 \ N/mm^2$

$\sigma_3 = -222.22 \ N/mm^2$

$R_1 = -2.6667\times10^5 N$

$R_4 = -1.333\times10^5 N$

$R_2 = 0$

$R_3 = 0$

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13. (a)

Given :

$u_1 = 0.005\,mm$

$u_2 = 0.0\,mm$

$u_3 = 0.005\,mm$

$x_1 = 5\,mm$

$x_2 = 15\,mm$

$x_3 = 25\,mm$

$v_1 = 0.002\,mm$

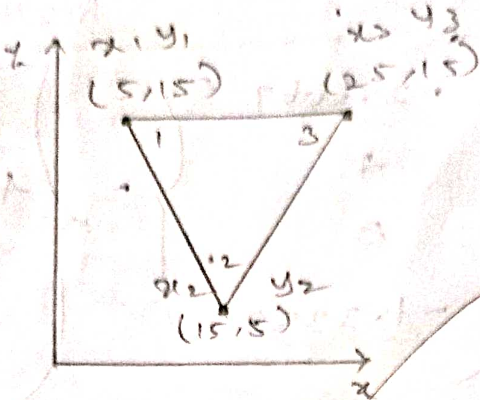
$v_2 = 0.0\,mm$

$v_3 = 0.0\,mm$

$y_1 = 15\,mm$

$y_2 = 5\,mm$

$y_3 = 15\,mm$



$E = 70\,GPa = 70\times10^3\,N/mm^2$

$\nu = 0.3$

$t = 1\,mm$

Soln :

$$A = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 5 & 15 \\ 1 & 15 & 5 \\ 1 & 25 & 15 \end{vmatrix}$$

$$= \frac{1}{2}\times[1\times(15\times15-5\times25)-5(1\times15-5\times1)+15(1\times25-$$

$$\boxed{A = 100\,mm^2}$$

$$[B] = \frac{1}{2A}\begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

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$q_1 = y_2 - y_3 = 5 - 15 = -10$

$q_2 = y_3 - y_1 = 15 - 15 = 0$

$q_3 = y_1 - y_2 = 15 - 5 = 10$

$r_1 = x_3 - x_2 = 25 - 15 = 10$

$r_2 = x_1 - x_3 = 5 - 25 = -20$

$r_3 = x_2 - x_1 = 15 - 5 = 10$

$$[B] = \frac{1}{2 \times 100} \begin{bmatrix} -10 & 0 & 0 & 0 & 10 & 0 \\ 0 & 10 & 0 & -20 & 0 & 10 \\ 10 & -10 & -20 & 0 & 10 & 10 \end{bmatrix}$$

$$[D] = \frac{E}{(1+v)(1-2v)} \begin{bmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & \frac{1-2v}{2} \end{bmatrix}$$

$$= \frac{90 \times 10^3}{(1+0.3)(1-2\times 0.3)} \begin{bmatrix} 1-0.3 & 0.3 & 0 \\ 0.3 & 1-0.3 & 0 \\ 0 & 0 & \frac{1-2\times 0.3}{2} \end{bmatrix}$$

$$[D] = 126.923 \times 10^3 \begin{bmatrix} 3.5 & 1.5 & 0 \\ 1.5 & 3.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$2\lambda^3 w$

$$[D][B] = 26.923\times10^{3}\begin{bmatrix}3.5 & 1.5 & 0\\ 1.5 & 3.5 & 0\\ 0 & 0 & 1\end{bmatrix}\times\frac{1}{2\times100}\begin{bmatrix}-10 & 0 & 0 & 0 \\ 0 & 10 & 0 & -20\\ 10 & -10 & -20 & 0\end{bmatrix}$$

$$[D][B] = 134.615\begin{bmatrix}-35 & 15 & 0 & -30 & 35 & 15\\ -15 & 35 & 0 & -70 & 15 & 35\\ 10 & -10 & -20 & 0 & 10 & 10\end{bmatrix}$$

$$\{\sigma\} = [D][B]\{u\}$$

$$= [D][B]\begin{Bmatrix}u_1\\ v_1\\ u_2\\ v_2\\ u_3\\ v_3\end{Bmatrix}$$

$$= 134.615\begin{bmatrix}-35 & 15 & 0 & -30 & 35 & 15\\ -15 & 35 & 0 & -70 & 15 & 35\\ 10 & -10 & -20 & 0 & 10 & 10\end{bmatrix}\times\begin{bmatrix}0.005\\ 0.002\\ 0.0\\ 0.0\\ 0.005\\ 0.0\end{bmatrix}$$

$$(\sigma) = \begin{Bmatrix}4.038\\ 9.423\\ 10.769\end{Bmatrix}$$

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{pmatrix} = \begin{Bmatrix}4.038\\ 9.423\\ 10.769\end{Bmatrix}$$

$\sigma_x = 4.038\ N/mm^2$

$\sigma_y = 9.423\ N/mm^2$

$\tau_{xy} = 10.769\ N/mm^2$

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Name: **Reg.No:**

$$\sigma_{max} = \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{4.038 + 9.423}{2} + \sqrt{\left(\frac{4.038 - 9.423}{2}\right)^2 + (10.769)^2}$$

$$\boxed{\sigma_1 = 17.83\ N/mm^2}$$

$$\sigma_{min} = \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{4.038 + 9.423}{2} - \sqrt{\left(\frac{4.038 - 9.423}{2}\right)^2 + (10.769)^2}$$

$$\boxed{\sigma_2 = -4.369\ N/mm^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$2\theta_p = \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = \tan^{-1}\left(\frac{2 \times 10.769}{4.038 - 9.423}\right)$$

$$2\theta_p = -75.96^\circ$$

$$\boxed{\theta_p = -37.98^\circ}$$

Result:

$\sigma_x = 4.038\ N/mm^2$

$\sigma_y = 9.423\ N/mm^2$

$\tau_{xy} = 10.769\ N/mm^2$

$\sigma_1 = 17.83\ N/mm^2$

$\sigma_2 = -4.369\ N/mm^2$

$\theta_p = -37.98^\circ$

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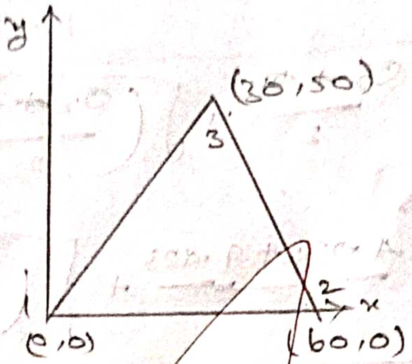
14. (a)

Given:



$r_1 = 0$ mm

$r_2 = 60$ mm

$r_3 = 30$ mm

$u_1 = 0.05$ mm

$u_2 = 0.02$ mm

$u_3 = 0$ mm

$E = 2.1\times10^5$ N/mm²

$\nu = 0.25$

$z_1 = 0$ mm

$z_2 = 0$ mm

$z_3 = 50$ mm

$w_1 = 0.03$ mm

$w_2 = 0.02$ mm

$w_3 = 0$ mm

Soln:

$$\{\sigma\} = [D][B]\{u\}$$

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \tau_{rz} \end{Bmatrix} = [D][B] \begin{Bmatrix} u_1 \\ w_1 \\ u_2 \\ w_2 \\ u_3 \\ w_3 \end{Bmatrix}$$

$$A = \frac{1}{2}\times B\times H = \frac{1}{2}\times60\times50 = \boxed{1500\ \text{mm}^2}$$

$$r = \frac{r_1+r_2+r_3}{3} = \frac{0+60+30}{3}$$

$$\boxed{r = 30\ \text{mm}}$$

$$z = \frac{z_1+z_2+z_3}{3} = \frac{0+0+50}{3}$$

$$\boxed{z = 16.667\ \text{mm}}$$

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$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

$$[D] = \frac{2.1\times10^5}{(1+0.25)[1-(2\times0.25)]} \begin{bmatrix} 0.75 & 0.25 & 0.25 & 0 \\ 0.25 & 0.75 & 0.25 & 0 \\ 0.25 & 0.25 & 0.75 & 0 \\ 0 & 0 & 0 & 0.25 \end{bmatrix}$$

$$[D] = 84\times10^3 \begin{bmatrix} 3 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[B] = \frac{1}{2A} \begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ \frac{\alpha_1}{r}+\beta_1+\frac{\gamma_1 z}{r} & 0 & \frac{\alpha_2}{r}+\beta_2+\frac{\gamma_2 z}{r} & 0 & \frac{\alpha_3}{r}+\beta_3+\frac{\gamma_3 z}{r} & 0 \\ 0 & \gamma_1 & 0 & \gamma_3 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 \end{bmatrix}$$

$$\alpha_1 = r_2 z_3 - r_3 z_2 = (60\times50)(30\times0)$$

$$\boxed{\alpha_1 = 3000\ mm^2}$$

$$\alpha_2 = r_3 z_1 - r_1 z_3 = (30\times0) - (0\times50)$$

$$\boxed{\alpha_2 = 0}$$

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$\alpha_3 = r_1 z_2 - r_2 z_1 = (0 \times 0) - (60 \times 0)$

$\boxed{\alpha_3 = 0}$

$\beta_1 = z_2 - z_3 = 0 - 50$

$\boxed{\beta_1 = -50mm}$

$\beta_2 = z_3 - z_1 = 50 - 0$

$\boxed{\beta_2 = 50mm}$

$\beta_3 = z_1 - z_2 = 0 - 0$

$\boxed{\beta_3 = 0}$

$\gamma_1 = r_3 - r_2 = 30 - 60$

$\boxed{\gamma_1 = -30mm}$

$\gamma_2 = r_1 - r_3 = 0 - 30$

$\boxed{\gamma_2 = -30mm}$

$\gamma_3 = r_2 - r_1 = 60 - 0$

$\boxed{\gamma_3 = 60mm}$

$\frac{\alpha_1}{r} + \beta_1 + \frac{\gamma_1 z}{r} = \frac{3000}{30} + (-50) + \frac{(-30 \times 16.667)}{30} = 33.33mm$

$\frac{\alpha_2}{r} + \beta_2 + \frac{\gamma_2 z}{r} = 0 + 50 + \left(\frac{-30 \times 16.667}{30}\right) = 33.33mm$

$\frac{\alpha_3}{r} + \beta_3 + \frac{\gamma_3 z}{r} = 0 + 0 + \left(\frac{60 \times 16.667}{30}\right) = 33.33 mm$

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Name: **Reg.No:**

$$[B] = \frac{1}{2\times1500}\begin{bmatrix} -50 & 0 & 50 & 0 & 0 & 0 \\ 33.33 & 0 & 33.33 & 0 & 33.33 & 0 \\ 0 & -30 & 0 & -30 & 0 & 60 \\ -30 & -50 & -30 & 50 & 60 & 0 \end{bmatrix}$$

$$[D][B] = 84\times10^3 \begin{bmatrix} 3 & 1 & 1 & 0 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times 3.333\times10^{-4}$$

$$\begin{bmatrix} -50 & 0 & 50 & 0 & 0 & 0 \\ 33.33 & 0 & 33.33 & 0 & 33.33 & 0 \\ 0 & -30 & 0 & -30 & 0 & 60 \\ -30 & -50 & -30 & -50 & 60 & 0 \end{bmatrix}$$

$$[D][B] = 28 \begin{bmatrix} -116.67 & -30 & 183.33 & -30 & 33.33 & 60 \\ 49.99 & -30 & 149.9 & -30 & 99.99 & 60 \\ -16.67 & -90 & 83.33 & -90 & 33.33 & 180 \\ -30 & -50 & -30 & 50 & 60 & 0 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \sigma_{rz} \end{Bmatrix} = 28 \begin{bmatrix} -116.67 & -30 & 183.3 & -30 & 33.33 & 60 \\ 49.99 & -30 & 149.9 & -30 & 99.9 & 60 \\ -16.67 & -90 & 83.33 & -90 & 33.33 & 180 \\ -30 & -50 & -30 & 50 & 60 & 0 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \sigma_{rz} \end{Bmatrix} = 28 \begin{Bmatrix} -3.666 \\ 4 \\ -3.666 \\ -2.6 \end{Bmatrix}$$

$\sigma_r = -102.65 \ N/mm^2$

$\sigma_\theta = 112 \ N/mm^2$

$\sigma_z = -102.65 \ N/mm^2$

$\sigma_{rz} = -72.8 \ N/mm^2$

Result:

$\sigma_r = -102.65 \ N/mm^2$

$\sigma_\theta = 112 \ N/mm^2$

$\sigma_z = -102.65 \ N/mm^2$

$\sigma_{rz} = -72.8 \ N/mm^2$

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ADDITIONAL SHEET

Name: Reg.No:

15. (b)

Given:

$$I = \int_{-1}^{1}\int_{-1}^{1} (2x^2 + 3xy + 4y^2)\,dx\,dy$$

$$f(x,y) = (2x^2 + 3xy + 4y^2)$$

soln:

$2n-1 = 2$

$n = 1.5 = 2$

$\boxed{n=2}$

$x_1 = 0.57735$ $y_1 = 0.57735$

$x_2 = -0.57735$ $y_2 = -0.57735$

$w_1 = 1$

$w_2 = 1$

$$\int_{-1}^{1}\int_{-1}^{1} f(x,y)\,dx\,dy = w_1^2 f(x_1,y_1) + w_1 w_2 f(x_1,y_2) + w_2 w_1 f(x_2,y_1) + w_2^2 f(x_2,y_2)$$

we know that,

$$f(x,y) = (2x^2 + 3xy + 4y^2)$$

$$w_1^2 f(x_1,y_1) = w_1^2 (2x_1^2 + 3x_1y_1 + 4y_1^2)$$

$$= 1^2(2(0.57735)^2 + 3(0.57735)(0.57735) + 4(0.57735)^2]$$

$w_1^2 f(x_1, y_1) = 3$

$w_1 w_2 f(x_1, y_2) = w_1 w_2 (2x_1^2 + 3x_1 y_2 + 4y_2^2)$

$= 1 \times 1 [2(0.57735)^2 + 3(0.57735) \cdot (-0.57735)$

$+ 4(-0.57735)^2]$ 6

$w_1 w_2 f(x_1, y_2) = w_1 w_2 (2x_1^2 + 3x_1 y_2 + 4y_2^2)$

$= 1 \times 1 [2(0.57735)^2 + 3(0.57735)(-0.57735)$

$4(-0.57735)^2]$

$w_1 w_2 f(x_1, y_1) = 1$

$w_2 w_1 f(x_2, y_1) = w_2 w_1 (2x_2^2 + 3x_2 y_1 + 4y_1^2)$

$= 1 \times 1 [2(-0.57735)^2 + 3(-0.57735)(0.57735)$

$4(0.57735)^2]$.

$w_2 w_1 f(x_2, y_1) = 1$

$w_2^2 f(x_2 y_2) = w_2^2 (2x_2^2 + 3x_2 y_2 + 4y_2^2)$

$= 1^2 [2(-0.57735)^2 + 3(-0.57735)(-0.57735)$.

$4(-0.57735)^2]$

$w_2^2 f(x_2, y_2) = 3$

$$\int_{-1}^{1}\int_{-1}^{1} (2x^2 + 3xy + 4y^2)\,dx\,dy = 3+1+1+3 = 8$$

$$\boxed{\int_{-1}^{1}\int_{-1}^{1} (2x^2 + 3xy + 4y^2)\,dx\,dy = 8}$$

Result:

$$\int_{-1}^{1}\int_{-1}^{1} (2x^2 + 3xy + 4y^2)\,dx\,dy = 8$$

ADDITIONAL SHEET

Name: Reg.No:

16. (a)

Soln:

Step 1:

$$\begin{bmatrix} 2 & 4 & 2 & : & 15 \\ 2 & 1 & 2 & : & -5 \\ 4 & 1 & -2 & : & 0 \end{bmatrix}$$

Step 2:

$$\begin{bmatrix} 2 & 4 & 2 & : & 15 \\ 2 & 1 & 2 & : & -5 \\ 4 & 1 & -2 & : & 0 \end{bmatrix}$$

Step 3:

$$\begin{bmatrix} 4 & 1 & -2 & : & 0 \\ 2 & 1 & 2 & : & -5 \\ 2 & 4 & 2 & : & 15 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

Step 4:

$$\begin{bmatrix} 1 & 1/4 & -2/4 & : & 0 \\ 2 & 1 & 2 & : & -5 \\ 2 & 4 & 2 & : & 15 \end{bmatrix} \quad R_1 \rightarrow R_1/4$$

Step 5:

$$\begin{bmatrix} 1 & 1/4 & -2/4 & : & 0 \\ 0 & 2/4 & 3 & : & -5 \\ 0 & 14/4 & 3 & : & +15 \end{bmatrix} \quad \begin{matrix} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 2R_3 \end{matrix}$$

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Step 6:

$$\left[\begin{array}{ccc:c} 1 & \frac{1}{4} & \frac{-2}{4} & 0 \\ 0 & \frac{2}{4} & 3 & -5 \\ 0 & \frac{14}{4} & 3 & 15 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & \frac{1}{4} & \frac{-2}{4} & 0 \\ 0 & \frac{14}{4} & 3 & 15 \\ 0 & \frac{2}{4} & 3 & -5 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

R_2 multiple by $\frac{4}{14}$

$$\left[\begin{array}{ccc:c} 1 & \frac{1}{4} & \frac{-2}{4} & 0 \\ 0 & 1 & \frac{12}{14} & \frac{60}{14} \\ 0 & \frac{2}{4} & 3 & -5 \end{array}\right] \quad R_2 \rightarrow R_2 \times \frac{4}{14}$$

$$\left[\begin{array}{ccc:c} 1 & \frac{1}{4} & \frac{-2}{4} & 0 \\ 0 & 1 & \frac{12}{14} & \frac{60}{14} \\ 0 & 0 & \frac{18}{7} & \frac{-50}{7} \end{array}\right]$$

ADDITIONAL SHEET

Name: **Reg.No:**

Step 7:

$$x_1 + \frac{1}{4}x_2 - \frac{2}{4}x_3 = 0$$

$$x_2 + \frac{12}{14}x_3 = \frac{60}{14}$$

$$\frac{18}{7}x_3 = \frac{-50}{7}$$

The equations can be solved,

$$\frac{18}{7}x_3 = \frac{-50}{7}$$

$$\boxed{x_3 = -2.77}$$

$$x_2 + \frac{12}{14}x_3 = \frac{60}{14}$$

$$\boxed{x_2 = 6.66}$$

$$x_1 + \frac{1}{4}x_2 - \frac{2}{4}x_3 = 0$$

$$\boxed{x_1 = -3.05}$$

Result:

$x_1 = -3.05$

$x_2 = 6.66$

$x_3 = -2.77$.